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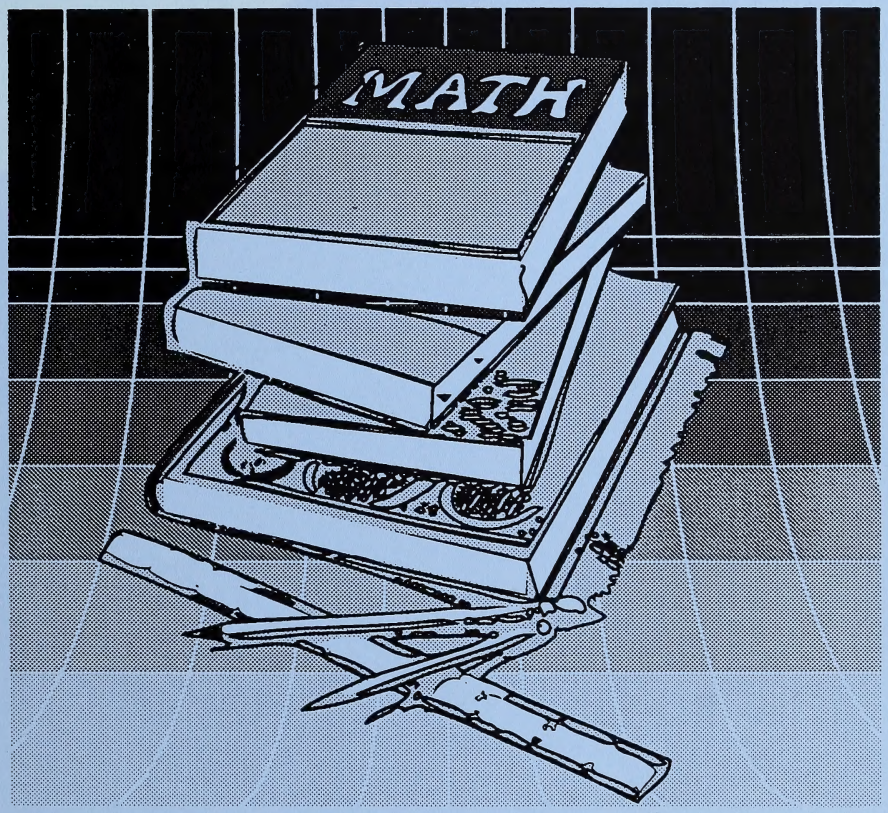
MATHEMATICS 20



LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta
EDUCATION

Mathematics 20

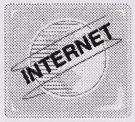
LEARNING FACILITATOR'S MANUAL

Alberta
EDUCATION

NOTE: This Mathematics 20 Learning Facilitator's Manual contains the Final Test and answers to the Final Test and to the teacher-assessed assignments; therefore, it should be kept securely by the teacher at all times. The Final Test should be accessible to students only in supervised situations.

Mathematics 20
Learning Facilitator's Manual
Modules 1–8
Alberta Distance Learning Centre
ISBN 0-7741-0004-4

This document is intended for	
Students	
Teachers (Mathematics 20)	✓
Administrators	
Parents	
General Public	
Other	



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Final Test : Teacher's Marking Guide

Final Test

Questionnaire

Introduction

A survey of these course materials will confirm that this new learning package has been specially designed for many kinds of teachers working in a variety of situations.

Which Category Do You Fit?

- ❒ Small Schools Teacher
  - ❒ inexperienced
  - ❒ experienced, but in other subject areas
  - ❒ experienced in mathematics but wanting to try a different approach
- ❒ Distance Learning Teacher
  - ❒ travelling to schools within the jurisdiction
  - ❒ using facsimile and teleconferencing to teach students within the area
- ❒ Larger Schools Teacher
  - ❒ inexperienced
  - ❒ experienced in mathematics but wanting to try a different approach

Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers regardless of their situation.

Advantages for Students

- incorporates a strong learner-centered philosophy
- promotes such qualities in the learner as autonomy, independence, flexibility
- is developed through media which suits the needs and circumstances of the learner
- reflects the experiential background of Alberta students
- opens up opportunities by overcoming barriers that result from geographical location
- promotes individualized learning, allowing learners to work at their own pace

Advantages for Teachers

- allows teachers maximum teaching time and minimizes preparation time
- includes different routes through the materials to suit different learners
- incorporates a wide range of teaching strategies, in particular those using independent and individual learning
- delivers curriculum designed by education specialists that reflects Alberta Education Program of Studies with an emphasis on Canadian content
- provides learning materials which are upwardly compatible with advanced educational technology

Does it sound like something you could use?

The student materials are not the only components designed for independent, guided instruction; so is this Learning Facilitator's Manual. It begins with an overview of the current Alberta Education Program of Studies for Mathematics. This summary is included for inexperienced teachers or those teachers who have found themselves teaching mathematics when their training is in other subject areas. This brief summary is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this new courseware and how they might want to use it in their classroom.

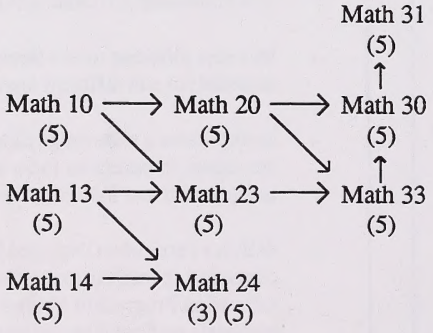
Beyond the introduction, the guide itself contains answers, models, explanations and other tips generated by the teachers who authored this course.

The courseware and LFM are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.

Overview of Program of Studies

A knowledge of mathematics is essential in many other areas of learning. With available information and the use of technology the emphasis will not be so much on memorizing formulae but on how to use mathematical information to solve problems. You, the learner, will be expected to construct, investigate, translate, discuss, analyze and formulate mathematics in a dynamic way.

The Mathematics 10 - 20 - 30 - 31 program is the academic series leading to university entrance. The Mathematics 13 - 23 - 33 program is the vocational series leading to technical institute entrance. The Mathematics 14 - 24 program is a general series suitable for students who have experienced difficulties in previous mathematics courses. The complete high school mathematics program is illustrated below along with recommended transfer points. The number in brackets indicates credits.



Your mathematics course is divided into units. Each unit covers a major area of mathematical study, and is divided further into specific content areas called topics. Each unit has its own Assignment Booklet which you complete and submit at the end of that unit.

Some of our distance learning courses are divided into sections within modules. If you receive letters or other materials from Alberta Correspondence School which refer to modules, please remember that a module is the same as a unit and a section is the same as a topic.

Module	=	Unit

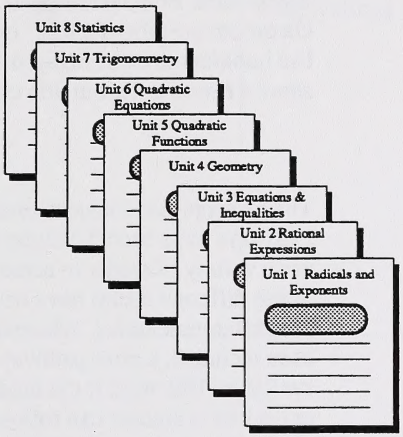
Section	=	Topic

Structure of the Learning Package

Basic Design

This new learning package involves many other components in addition to the Learning Facilitator's Manual.

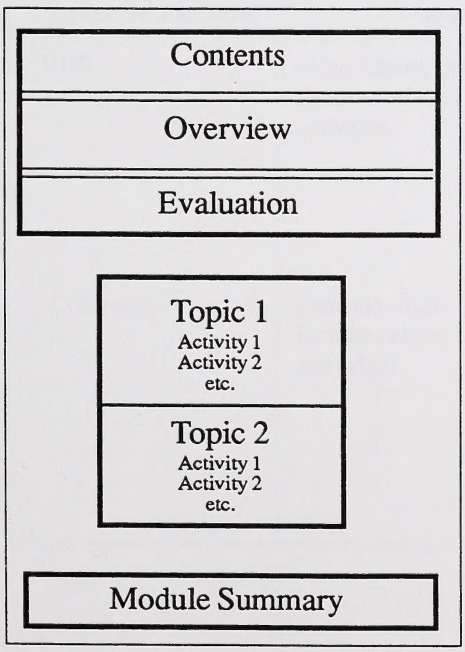
Units



The print components involve many booklets called units. These units contain guided activities that instruct students in a relevant realistic setting.

The units have been specially designed to promote such qualities in the learner as autonomy, independence and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, using advance, intermediate and post organizers and many other techniques to enable learners to learn on their own for at least some of the time.

The structure of the Unit Booklets follows a systematic design. Each unit begins with a detailed table of contents which shows the students all the main steps. It acts as an organizer for students. The overview introduces the unit title or theme. A graphic representation has been included to help visual learners and poor readers. The introduction also states the weightings of each assignment.

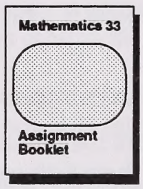


The body of the unit is made up of two or more closely related topics. Each topic contains student activities that develop skills and knowledge centered around a theme.

The activities may involve print, audio and video formats. At times, students and learning facilitators are allowed to choose the activity that best suits the student's needs and interests. Other activities such as the Extra Help and Extensions are optional pathways. This flexibility caters to each student's personal situation.

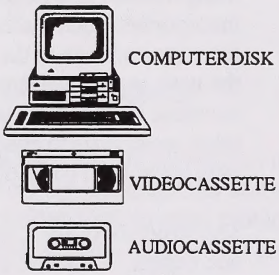
The summary focuses on the skills and strategies that the student has learned.

Assignment Booklet



Accompanying each unit is an Assignment Booklet. The activities in these booklets can be used for formative and for summative assessments. The students should complete the Assignment Booklet when they have thoroughly reviewed the unit materials. The Assignment Booklet has been designed for classroom use, for "faxing," or for mailing. **If the booklets are not being mailed, you should remove the outside cover.**

Media



The package also includes reference to media. Pathways have been developed so students can use a variety of media to achieve the objective. These different routes have been included to suit different learners. Wherever videos have been included, a print pathway is also available. This way, if the media isn't available or desired, a student can follow the print pathway and still successfully achieve the objective.

Materials, Media and Equipment

Mandatory Components

Equipment (Hardware)	Media	Materials
audiocassette player	prepared audiocassette (comes with learning package)	**Unit Booklets (8)** **Assignment Booklets (8)** **Geometry Set** **Scientific calculator**

Optional Components

Equipment (Hardware)	Media	Materials
VCR	videocassette (suggestions included as well as related activities)	
Computer	computer disks (suggestions included as well as related activities)	

Using This Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each unit, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

Open-Learning Classroom

Open-learning is the concept of opening up opportunities by overcoming barriers of time, pace and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual student's needs. But these efforts often failed, due to lack of time and lack of quality materials that conformed to Alberta specifications.

Due to advanced educational technology and improved Alberta-specific learning packages, a student-centered approach is now possible. Improved technology now allows us to provide support to learners, individually, regardless of their pace or location. A teacher cannot be in 28 places at one time offering guidance - media and a well-designed learning package can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

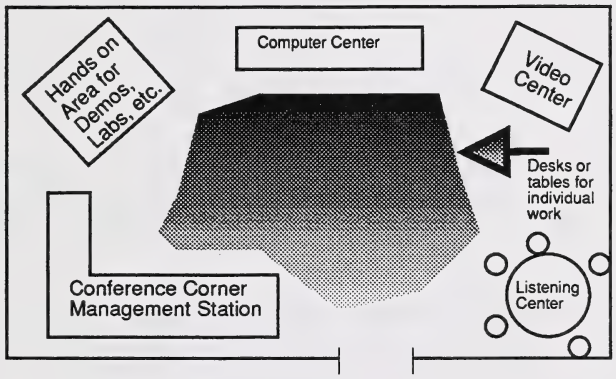
The key to a successful open-learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own, for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open-learning system runs smoothly.

The Key to a Successful Open-Learning System

1. Learning Package

The specially designed learning package needed for a successful open-learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all the components in the learning package available to students as needed.

If adequate numbers of media are available to satisfy the demand, a center can be established for specific media:



You may not have the luxury to have enough hardware to set up a permanent video or computer center in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every 3 - 5 days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

2. Support

Support is definitely a key element for any successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for each of the activities included in the Unit Booklet. High school mathematics units may have solutions, models, explanations, and guides included in the Appendix of every Unit Booklet. These are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking.

As the learning facilitator, you may be needed to offer more personal guidance to those students having difficulty, or you may need to reinforce the need for doing these activities carefully before attempting the assignments in the Assignment Booklet.

The activities include choices and pathways. If a student is having difficulty, you may need to encourage that student to work on all the choices rather than one. This would provide additional instruction and practice in a variety of ways.

Another form of support is routine contact with each individual. This might be achieved with a bi-weekly conference scheduled by you, or as students reach a certain point (e.g. after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This might involve small groups or a large group. It might be used to take advantage of something timely (e.g. election, eclipse, etc.) or something prescheduled like the demonstration of a process, or involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

3. Management

Good management of an open learning system is essential to the success of the program. The following areas need action to ensure that the system runs smoothly:

1. Scheduling, Distributing and Managing Resources - As discussed earlier, this may require a need for centers or a system for students to project and reserve the necessary resources.

2. Scheduling Students - Students and teachers should work together to establish goals, course completion timelines and daily timelines. Although students may push to continue for long periods of time (e.g. all morning), teachers should discourage this. Concentration, retention and motivation is improved by taking scheduled breaks.

3. Monitoring Student Progress - You will need to record when units are completed by each student. Your data might also include the projected date of completion if you are using a student contract approach.

Sample of a Student Progress Chart

Mathematics		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Final Test
Billy Adams	P									
	A									
Jane Bell	P									
	A									
Joe Bloom	P									
	A									
P = Projected Completion Date A = Actual Completion Date										

The student could keep a personal log as well.
Such tracking of data could be stored easily on a computer.

4. Recording Student Assessments - You will need to record the marks awarded to each student for work completed in each unit Assignment Booklet. The marks from these assignment booklets will contribute to a portion of the student's final mark. The final test will also be used, and other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria is, it should be made clear to all students at the beginning.

Sample of a Student's Assessment Chart

Mathematics	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Years Aver.	Final Test	Final Mark
Billy Adams	67	65	54	47	78	67	63	77	65		
Jane Bell	43	50	54	55	48	42	52	41	43		
Joe Bloom	65	65	66	68	67	70	66	61	66		

Letter grading could easily be substituted.

5. Recording Effectiveness of System - Keep ongoing records of how the system is working. This will help you in future planning.

Sample of a System Assessment Chart

Module 1			
Date	Unit Booklet	Assignment Booklet	Resources-Media

The Role of the Teacher in an Open-Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach it is necessary to move beyond the idea of a passive learner depending largely on the continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance and guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student-centered not teacher-centered. The teacher needs to facilitate learning by providing general support to the learner.

Evaluation

Evaluation is important to the development of every learner. Data-gathering, processing, and decision-making, both at the student and teacher level, serve as means of identifying strengths and weaknesses.

These specially-designed learning packages contain many kinds of informal and formal evaluation.

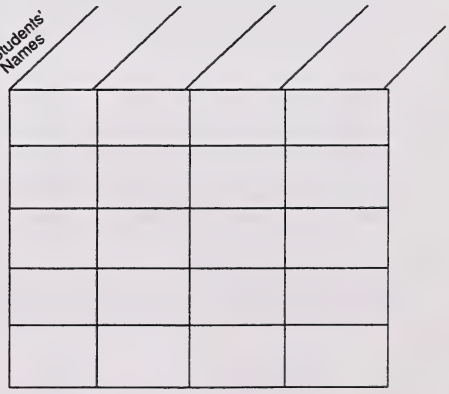
1. Observation

In the classroom the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist

1. Does the student approach the work in a positive manner?
2. Is the student struggling with the reading level?
3. Does the student make good use of time?
4. Does the student apply an appropriate study method?
5. Can the student use references effectively, etc.?



Observation may suggest a need for an individual interview with a student.

2. Individual Conferences

Individual conferences may be paced (scheduled) by the calendar or at certain points in the unit, or they may be set up only as needed or requested.

During these conferences teachers can determine the student's progress and they can assess the student's attitudes toward the subject, program, school and self, as well as his or her relationship with other students. With guided questions, the teacher can encourage oral self-assessment: the student can discuss his or her own strengths or weaknesses in regard to the particular topic, unit or subject area.

3. Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. By including activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude toward these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming an autonomous learner. By instructing, motivating, providing positive reinforcement and supervising systematically, the learning facilitator will help students develop a positive attitude toward their own progress.

For variations, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, have a peer assess other activities, and become directly involved in assessing the remainder of the activities.

When the activities have been assessed, the student should be directed to make corrections. This should be made clear to students right from the start. It is important to note the correct association between the question and response to clarify understanding, aid retention and be of use for study purposes.

Many of the activities include choices for students. If the student is having difficulty, more practice may be warranted, and the student may need to be encouraged to do more of the choices.

Each topic within a unit includes additional types of activities called Extra Help and Extensions. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

a. What steps are you taking to improve your understanding of this topic?

b. What method of study do you use most?

c. How do you organize your material to remember it?

d. What steps do you follow when doing an assignment in your Assignment Booklet?

e. What could you do to become an even better reader?

f. Do you have trouble following directions?

g. Did you enjoy this unit?

A chart or checklist could be used for recording responses.

4. Informal Evaluation: Assignments

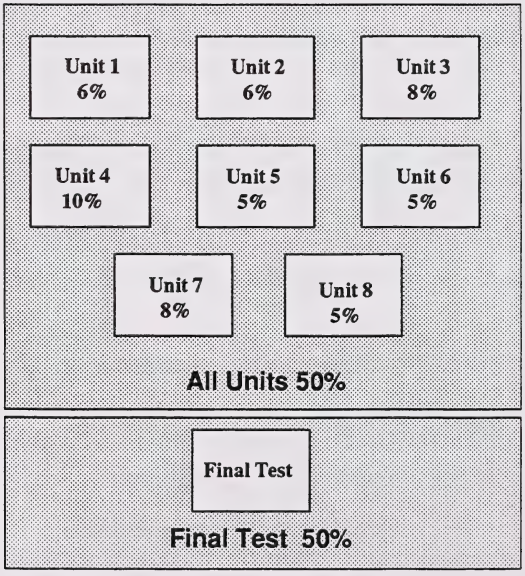
Informal evaluation such as the assignments included in each unit, are an invaluable aid to the teacher. They offer ongoing assessment information about the student's achievement and the behavior and attitudes that affect that achievement.

Each unit contains a separate booklet called the Assignment Booklet. This booklet assesses the knowledge or skills that the student has gained from the module. The student's mark for the module may be based solely on the outcome of learning evident in the Assignment Booklet; however, you may decide to establish a value for other variables such as attitude or effort. It is important that you establish at the beginning which outcomes will be evaluated , and that all students clearly understand what is expected.

5. Final Test

All LFM's include a formal final test which can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the unit booklets, gives the teacher precise information concerning what each student can or cannot do. Answers, explanations, and marking guides are also included.

The value of the final test and each unit is the decision of the classroom teacher. Following is a suggestion only.



Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be used with the class. After the materials have been explored, a discussion might include what the advantages and the disadvantages might be of learning independently or in small groups. The roles of the students and teacher should be analyzed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude toward learning autonomously.

Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. They are listed below:

1. understanding and using instructional materials (table of contents, index, list of illustrations, appendices, bibliography, glossary)
2. interpreting maps, graphs, charts
3. using reference materials
4. recognizing special symbols
5. using a scientific calculator

Other general skills are using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.

Reading Level

These course materials are largely print-based, but poorer readers need not be discouraged. It is important that you assure the students that these materials have been designed for easy reading. The authors have employed special strategies that lower and control the reading level. Some of them are

1. the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level
2. the integration of activities, examples and illustrations to break text into appropriate-sized chunks
3. the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post-organizers) to help give students a structure for incorporating new concepts

4. the recognition that vocabulary and concepts are basic to understanding content materials, and thus must be handled systematically (defined in context, through marginal notes, footnotes, and often in a specialized glossary)

5. the acknowledgement that background knowledge and experience plays a vital role in comprehension

6. the systematic inclusion of illustrations and optional videos to help poorer readers and visual learners, and audiocassettes and software also as an alternative to print-based learning

7. a variety of formats (paragraphs, lists, charts, etc.) to help poorer readers who do not absorb or retain main ideas easily in paragraph format

8. the inclusion of media pathways and activity choices to encourage an active rather than passive approach

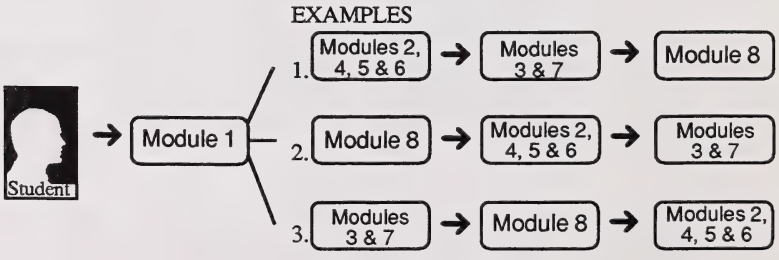
9. instruction in a meaningful setting rather than in a contrived, workbook style

10. using purposeful reading, viewing and doing to produce better interpretation of the course materials

11. the recognition that students need structured experiences when reading, viewing or listening instructional materials: developing pupil readiness, determining the purpose, providing guided instruction and feedback, rereading if necessary and extending. This structure closely resembles the reading process.

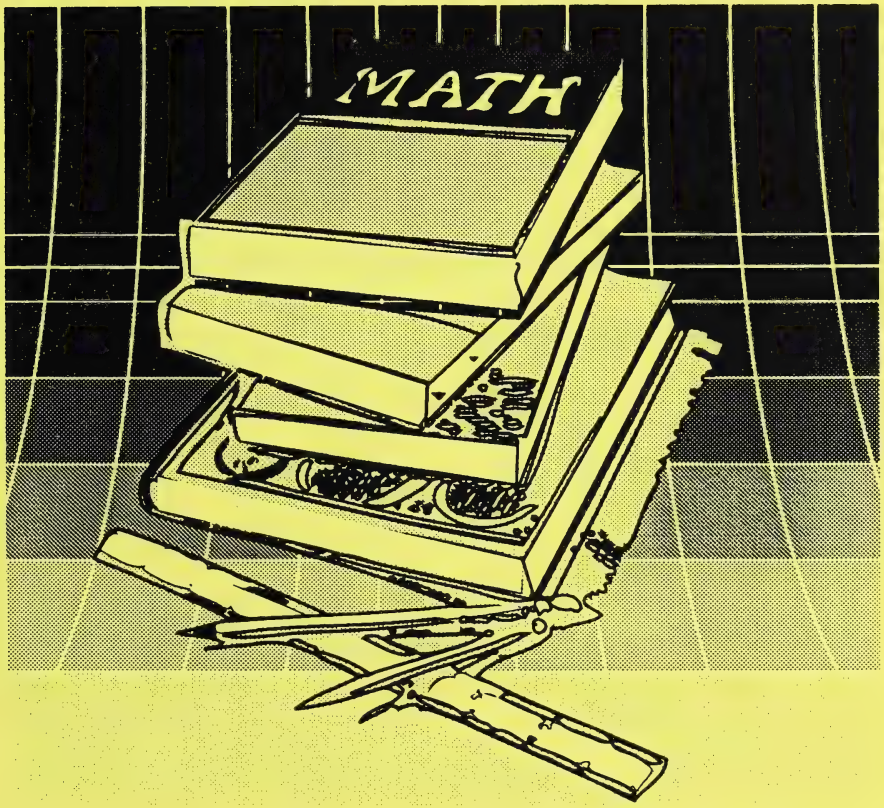
To help make the learning package more readable, you can begin in your unit preparation by reading (viewing, listening) to all the related materials that are going to be used. You need a solid background in order to assess and develop a background knowledge for students. The students' experiential bases may be assessed through brainstorming sessions concerning the topic, having students write down all the ideas they have about the topic, or by using visuals and guided questions to predict what the topic might be about.

It is recommended that you start with Unit 1 as this unit includes basic information on number systems. In many courses, further units can be tackled in any order. For solutions to all activities check in the Appendix of the respective unit. What will follow will be the solution to the Assignment Booklets, final test and the solutions to the final test.



UNIT 1

MATHEMATICS 20



LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

Topic 1: Radical and Exponential Forms

(6) 1. Transform the given exponential expressions to their radical equivalents.

a. $(2y)^{-\frac{3}{2}} = \frac{1}{(2y)^{\frac{3}{2}}} = \frac{1}{\left(\sqrt{2y}\right)^3}$

b. $(3)^{-\frac{1}{2}} = \frac{1}{(3)^{\frac{1}{2}}} = \frac{1}{\sqrt{3}}$

c. $x^{\frac{1}{5}} = \sqrt[5]{x}$

(8) 2. Transform the given radical expressions to their exponential equivalents.

a. $\sqrt[3]{x} = x^{\frac{1}{3}}$

b. $\left(\sqrt[3]{y}\right)^2 = y^{\frac{2}{3}}$

c. $\left(\sqrt[3]{-12}\right)^4 = (-12)^{\frac{4}{3}}$

d. $\sqrt[4]{(3x)^5} = (3x)^{\frac{5}{4}}$

(3) 3. Evaluate the following exponential expressions.

a. $(36)^{\frac{1}{2}} = \sqrt{36} = 6$

b. $\sqrt[4]{(6561)^3} = 9^3 = 729$

Topic 1

_____marks

Topic 2: Rational Exponents

(15) 1. Simplify the following and express the results with positive exponents.

a. $\dfrac{\left(xy^{\frac{1}{2}}z^{\frac{1}{3}}\right)^6}{\left(x^2y^2z^2\right)^{\frac{1}{2}}} = \dfrac{x^6y^3z^2}{xyz} = x^5y^2z$

b. $\dfrac{\left(x^{\frac{1}{2}}y^4\right)^{-2}}{\left(x^{\frac{3}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}\right)^2} = \dfrac{x^{-1}y^{-8}}{x^3yz} = \dfrac{1}{x^4y^9z}$

c. $\left[\dfrac{a^{\frac{1}{4}}b^{-2}}{a^{\frac{1}{2}}b^{-\frac{1}{4}}c^{\frac{1}{2}}}\right]^{-4} = \dfrac{a^{-1}b^8}{a^{-2}bc^{-2}} = ab^7c^2$

d. $\dfrac{\left(a^{\frac{1}{3}}b^{\frac{1}{3}}\right)^3}{\left(a^{\frac{3}{2}}b^2\right)^2} = \dfrac{ab}{a^3b^4} = \dfrac{1}{a^2b^3}$

e. $\left[\dfrac{4x^{\frac{1}{2}}y^{\frac{1}{3}}z^{-\frac{1}{2}}}{12x^{\frac{5}{4}}y^{\frac{1}{3}}z^0}\right]^{-4} = \left[\dfrac{12x^{\frac{5}{4}}y^{\frac{1}{3}}z^0}{4x^{\frac{1}{2}}y^{\frac{1}{3}}z^{-\frac{1}{2}}}\right]^4 = \left(3x^{\frac{3}{4}}z^{\frac{1}{2}}\right)^4 = 3^4x^{\frac{3}{4}\times4}z^{\frac{1}{2}\times4} = 81x^3z^2$

(3) 2. Simplify the following expressions.

a. $2\sqrt{3}\left(\sqrt{2}-\sqrt{3}\right)$

$= 2\sqrt{6}-6$

b. $\left(2\sqrt{3}-4\right)^2$

$= 12-16\sqrt{3}+16$

$= 28-16\sqrt{3}$

c. $-1\left(-3\sqrt{5}-1\right)^2$

$= -1\left(45+6\sqrt{5}+1\right)$

$= -45-6\sqrt{5}-1$

$= -46-6\sqrt{5}$ or $-6\sqrt{5}-46$

(4) 3. Evaluate:

a. $\sqrt[3]{\frac{8}{27}}$

$= \frac{2}{3}$

b. $\left(\frac{25}{9}\right)^3$

$= \frac{15625}{729}$

$= 21\frac{316}{729}$

$= 21.43347051$

c. $\left(\sqrt{16}\right)^3$

$= 4^3$

$= 64$

d. $\sqrt[3]{-27}$

$= -3$

Topic 2

_____marks

Topic 3: Operations on Radicals

(12) 1. Perform the indicated operations and express your answers using positive fractional exponents only.

a. $a^{\frac{1}{2}} \times a^{\frac{1}{4}}$

$= a^{\frac{1}{2}+\frac{1}{4}}$

$= a^{\frac{2}{4}+\frac{1}{4}}$

$= a^{\frac{3}{4}}$

b. $2a^{-\frac{1}{2}} \times a^{\frac{3}{4}} \times 2a^{-\frac{1}{4}}$

$= 4a^{-\frac{1}{2}+\frac{3}{4}+-\frac{1}{4}}$

$= 4a^{\frac{-2+3-1}{4}}$

$= 4a^{0}$

$= 4$

c. $c^{\frac{3}{8}} \div c^{\frac{1}{8}}$

$= c^{\frac{3}{8}-\frac{1}{8}}$

$= c^{\frac{2}{8}}$

$= c^{\frac{1}{4}}$

d. $12b^{\frac{1}{4}}c^{-\frac{2}{3}} \div 4b^{-\frac{3}{4}}c^{-\frac{4}{3}}$

$= (12 \div 4)b^{\frac{1}{4}+\frac{3}{4}}c^{-\frac{2}{3}+\frac{4}{3}}$

$= 3b^{\frac{4}{4}}c^{\frac{2}{3}}$

$= 3bc^{\frac{2}{3}}$

e. $\left(a^{\frac{1}{2}}b^{-\frac{1}{3}}\right)^{2}$

$= a^{1}b^{-\frac{2}{3}}$

$= \dfrac{a}{b^{\frac{2}{3}}}$

f. $\left[\dfrac{54a^{-\frac{1}{3}}}{2b^{-3}}\right]^{-\frac{1}{3}}$

$= \left[\dfrac{2b^{-3}}{54a^{-\frac{1}{3}}}\right]^{\frac{1}{3}}$

$= \left[\dfrac{b^{-3}}{27a^{-\frac{1}{3}}}\right]^{\frac{1}{3}}$

$= \dfrac{b^{-1}}{3a^{-\frac{1}{9}}}$

$= \dfrac{a^{\frac{1}{9}}}{3b}$

(2) 2. Circle the correct answer. Show how you obtained your answer.

$$\sqrt{\frac{11}{2}} =$$

A. $\frac{11}{2}$

B. $\frac{\sqrt{22}}{4}$

C. $\frac{11}{4}$

(D.) $\frac{\sqrt{22}}{2}$

$$\sqrt{\frac{11}{2}} = \frac{\sqrt{11}}{\sqrt{2}}$$
$$= \frac{\sqrt{11} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$
$$= \frac{\sqrt{22}}{2}$$

(2) 3. Circle the correct answer. Show how you obtained your answer.

$$\frac{1}{\sqrt{2}} =$$

(A.) $\frac{\sqrt{2}}{2}$

B. $\frac{\sqrt{2}}{4}$

C. 1

D. $\frac{1}{2}$

$$\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

(4) 4. Simplify the following by rationalizing the denominators.

a. $\sqrt{\frac{17}{2}}$

$= \frac{\sqrt{17} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$= \frac{\sqrt{34}}{2}$

b. $\sqrt{\frac{17}{3}}$

$= \frac{\sqrt{17} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \frac{\sqrt{51}}{3}$

(5) 5. The length d of the diagonal of a rectangle is given by the formula $d = \sqrt{a^2 + b^2}$, where a and b represent the lengths of the sides of the rectangle. If the length of the diagonal is 49 cm and the length of side b is 10 cm, determine the length of side a (to the nearest hundredth).

$$d = \sqrt{a^2 + b^2}$$
$$49 = \sqrt{a^2 + (10)^2}$$
$$49 = \sqrt{a^2 + 100}$$
$$(49)^2 = \left(\sqrt{a^2 + 100}\right)^2$$
$$2401 = a^2 + 100$$
$$a^2 = 2301$$
$$\sqrt{a^2} = \sqrt{2301}$$
$$a \doteq 47.9687$$
$$a \doteq 47.97$$

The length of side a is approximately 47.97 cm.

(3) 6. Write the given expressions as entire radicals.

a. $2k\sqrt{3k} = \sqrt{12k^3}$

b. $x^2\sqrt[3]{y^4} = \sqrt[3]{x^6y^4}$

(4) 7. Write the following expressions in simplest radical form.

a. $\sqrt{18}$
$= 3\sqrt{2}$

b. $\sqrt[3]{16}$
$= 2\sqrt[3]{2}$

(4) 8. Simplify the following radical expressions.

a. $5\sqrt[3]{64x} - 7\sqrt[3]{64x}$
$= 20\sqrt[3]{x} - 28\sqrt[3]{x}$
$= -8\sqrt[3]{x}$

b. $\sqrt[3]{x} - 2\sqrt[3]{x} - 4\sqrt[3]{x}$
$= -5\sqrt[3]{x}$

c. $5\sqrt[3]{64ac} - \sqrt[3]{125ac}$
$= 20\sqrt[3]{ac} - 5\sqrt[3]{ac}$
$= 15\sqrt[3]{ac}$

d. $2\sqrt{18a^2bc} - 3a\sqrt{2bc}$
$= 6a\sqrt{2bc} - 3a\sqrt{2bc}$
$= 3a\sqrt{2bc}$

Topic 3

_____marks

Topic 4: Radical Equations

(8) 1. Solve the radical expressions for the following variables.

a. $\sqrt[3]{b} = -6$

$\left(\sqrt[3]{b}\right)^3 = (-6)^3$

$b = -216$

b. $\sqrt[3]{v} = -8$

$\left(\sqrt[3]{v}\right)^3 = (-8)^3$

$v = -512$

c. $3\sqrt{3x} = 2\sqrt{3x} + 6$

$3\sqrt{3x} - 2\sqrt{3x} = 6$

$\sqrt{3x} = 6$

$\left(\sqrt{3x}\right)^2 = 6^2$

$3x = 36$

$x = 12$

d. $4\sqrt{3x} = 2\sqrt{3x} + 6$

$4\sqrt{3x} - 2\sqrt{3x} = 6$

$2\sqrt{3x} = 6$

$\left(2\sqrt{3x}\right)^2 = 6^2$

$12x = 36$

$x = 3$

(4) 2. Solve and verify the roots of the following radical expressions.

a. $\sqrt{x+6} - 15 = 14$

$\sqrt{x+6} = 29$

$\left(\sqrt{x+6}\right)^2 = 29^2$

$x + 6 = 841$

$x = 835$

The root is 835.

Check for $x = 835$.

LS	RS
$\sqrt{x+6} - 15$	14
$= \sqrt{835+6} - 15$	
$= \sqrt{841} - 15$	
$= 29 - 15$	
$= 14$	

LS = RS

b. $\left(\sqrt[3]{10x-1}\right) - 6 = 4$

$\left(\sqrt[3]{10x-1}\right) = 10$

$\left(\sqrt[3]{10x-1}\right)^3 = 10^3$

$10x - 1 = 1000$

$10x = 1001$

$x = 100.1$

The root is 100.1.

Check for $x = 100.1$.

LS	RS
$\left(\sqrt[3]{10x-1}\right) - 6$	4
$= \left(\sqrt[3]{10(100.1)-1}\right) - 6$	
$= \left(\sqrt[3]{1001-1}\right) - 6$	
$= \sqrt[3]{1000} - 6$	
$= 10 - 6$	
$= 4$	

LS = RS

(3) 3. When the cube root of the difference of a number and 2 is increased by 5, the result is 7. What is the number?

Let the number be x.

$$\sqrt[3]{x-2}+5=7$$
$$\sqrt[3]{x-2}=2$$
$$\left(\sqrt[3]{x-2}\right)^3=2^3$$
$$x-2=8$$
$$x=10$$

The number is 10.

Check for $x = 10$.

LS	RS
$\sqrt[3]{x-2}+5$	7
$=\sqrt[3]{10-2}+5$	
$=\sqrt[3]{8}+5$	
$=2+5$	
$=7$	

LS = RS

(3) 4. When two times the square root of the difference of a number and 5 is subtracted from itself, the result is 5. What is the number?

Let the number be x.

$$x-2\sqrt{x-5}=5$$
$$-2\sqrt{x-5}=5-x$$
$$\left(-2\sqrt{x-5}\right)^2=(5-x)^2$$
$$4(x-5)=25-10x+x^2$$
$$4x-20=25-10x+x^2$$
$$x^2-14x+45=0$$
$$(x-9)(x-5)=0$$
$$x-9=0 \text{ or } x-5=0$$
$$x=9 \qquad x=5$$

Check for $x = 9$.

LS	RS
$x-2\sqrt{x-5}$	5
$=9-2\sqrt{9-5}$	
$=9-2\sqrt{4}$	
$=9-2(2)$	
$=9-4$	
$=5$	

LS = RS

Check for $x = 5$.

LS	RS
$x-2\sqrt{x-5}$	5
$=5-2\sqrt{5-5}$	
$=5-2\sqrt{0}$	
$=5-0$	
$=5$	

LS = RS

The number is 5 or 9.

(3) 5. When two divided by the square root of the sum of a number and one is added to the square root of the sum of the same number and one, the result is the same as the square root of the sum of the same number and six.
Find the number.

Let the number be x.

$$\sqrt{x+1}+\frac{2}{\sqrt{x+1}}=\sqrt{x+6}$$

$$\left(\sqrt{x+1}\right)\left(\sqrt{x+1}\right)+\left(\sqrt{x+1}\right)\left(\frac{2}{\sqrt{x+1}}\right)=\left(\sqrt{x+1}\right)\left(\sqrt{x+6}\right)$$

$$x+1+2=\sqrt{x^2+7x+6}$$

$$x+3=\sqrt{x^2+7x+6}$$

$$(x+3)^2=\left(\sqrt{x^2+7x+6}\right)^2$$

$$x^2+6x+9=x^2+7x+6$$

$$x^2-x^2+6x-7x=6-9$$

$$-x=-3$$

$$x=3$$

Check for $x = 3$.

LS	RS
$\sqrt{x+1}+\frac{2}{\sqrt{x+1}}$	$\sqrt{x+6}$
$=\sqrt{3+1}+\frac{2}{\sqrt{3+1}}$	$=\sqrt{3+6}$
$=\sqrt{4}+\frac{2}{\sqrt{4}}$	$=\sqrt{9}$
$=2+\frac{2}{2}$	$=3$
$=2+1$	
$=3$	

LS = RS

The number is 3.

④ 6. One half of the square root of the sum of a number and $\frac{1}{4}$ is equal to $\frac{1}{6}$ of the square root of the product of the number and 12. What is the number?

Let x be the number.

$$\frac{1}{2}\sqrt{x+\frac{1}{4}} = \frac{1}{6}\sqrt{12x}$$

$$12\left(\frac{1}{2}\sqrt{x+\frac{1}{4}}\right) = 12\left(\frac{1}{6}\sqrt{12x}\right)$$

$$6\sqrt{x+\frac{1}{4}} = 2\sqrt{12x}$$

$$\left(6\sqrt{x+\frac{1}{4}}\right)^2 = \left(2\sqrt{12x}\right)^2$$

$$36\left(x+\frac{1}{4}\right) = 4(12x)$$

$$36x+9 = 48x$$

$$48x-36x = 9$$

$$12x = 9$$

$$x = \frac{9}{12}$$

$$x = \frac{3}{4}$$

The number is $\frac{3}{4}$.

Check for $x = \frac{3}{4}$.

LS	RS
$\frac{1}{2}\sqrt{x+\frac{1}{4}}$	$\frac{1}{6}\sqrt{12x}$
$= \frac{1}{2}\sqrt{\frac{3}{4}+\frac{1}{4}}$	$= \frac{1}{6}\sqrt{12\left(\frac{3}{4}\right)}$
$= \frac{1}{2}\sqrt{1}$	$= \frac{1}{6}\sqrt{9}$
$= \frac{1}{2}(1)$	$= \frac{1}{6}(3)$
$= \frac{1}{2}$	$= \frac{3}{6}$
	$= \frac{1}{2}$

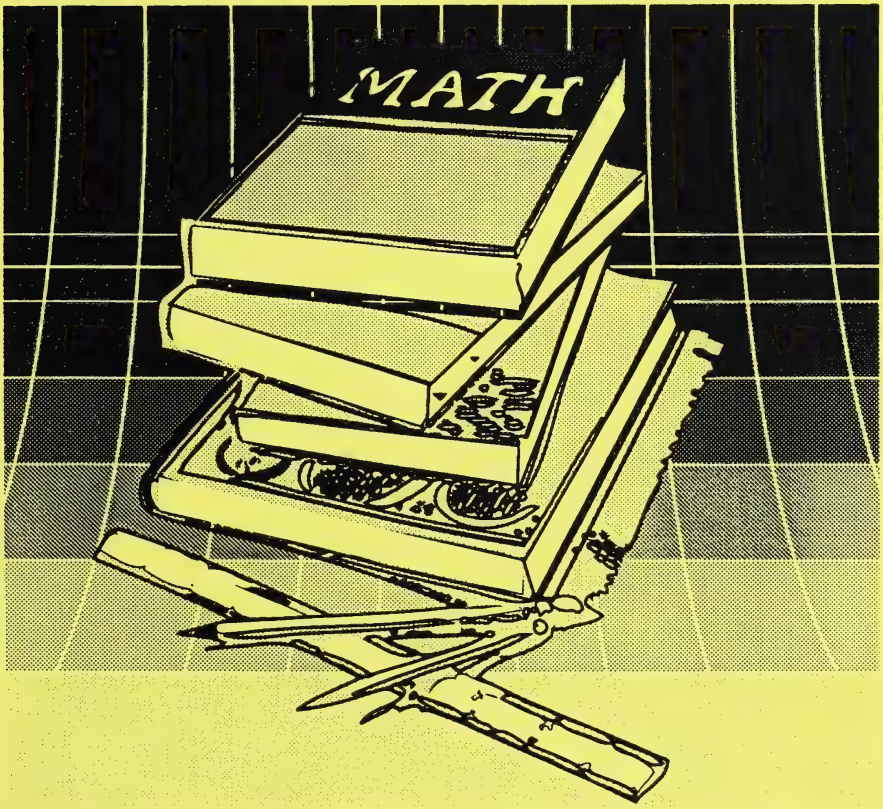
LS = RS

Topic 4

_____marks

UNIT 2

MATHEMATICS 20



LEARNING FACILITATOR'S MANUAL

Distance Learning

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EDUCATION

Topic 1: Nonpermissible Values

(5)

1. Identify the rational expressions. If the expression is a rational expression, write **R**. If the expression is not a rational expression, write **N**.

   a. $\frac{1}{3}$ **R**

   b. x^{-3} **R**

   c. $\frac{7x+2}{x^2-1}$ **R**

   d. $2\sqrt{x}+3$ **N**

   e. $x-\frac{1}{x}+4$ **R**

(10)

2. Determine the nonpermissible values of the variables in each of the following rational expressions.

   a. $\frac{3}{x}$ $x \neq 0$

   b. $\frac{y}{xy}$ $x \neq 0 \quad y \neq 0$

   c. $\frac{x-3}{x+4}$ $x \neq -4$

   d. $\frac{x-5}{2x^2-5x-7}$ $x \neq -1 \quad x \neq \frac{7}{2} \quad 2x^2-5x-7=(2x-7)(x+1)$

   e. $\frac{3}{x(x-1)(x+4)}$ $x \neq 0 \quad x \neq 1 \quad x \neq -4$

Topic 1

_____ marks

Topic 2: Simplifying Rational Expressions

1. Simplify each of the following:

(5) a. $\dfrac{14x^2y^5}{21x^3y^4} = \dfrac{2y}{3x}$

(5) b. $\dfrac{6x^2+2x}{6x^3-7x^2-3x} = \dfrac{2\overset{1}{\cancel{x}}(\overset{1}{\cancel{3x+1}})}{\underset{1}{\cancel{x}}(\underset{1}{\cancel{3x+1}})(2x-3)}$

$= \dfrac{2}{2x-3}$

(5) c. $\dfrac{3x^2-2x-5}{9x^2-25} = \dfrac{\overset{1}{\cancel{(3x-5)}}(x+1)}{\underset{1}{\cancel{(3x-5)}}(3x+5)}$

$= \dfrac{x+1}{3x+5}$

(5) 2. A bag of candy has a mass of $(15x^3-2x^2-x)$ g. It contains $(3x-1)$ pieces of candy. Find an expression for the average mass of each piece of candy.

$$\frac{15x^3-2x^2-x}{3x-1}$$
$$= \frac{x(5x+1)(\overset{1}{\cancel{3x-1}})}{\underset{1}{\cancel{(3x-1)}}}$$
$$= x(5x+1) \text{ g}$$

The average mass of a piece of candy is $x\,(5x + 1)$ g.

Topic 2

_____ marks

Topic 3: Multiplying and Dividing

(2) 1. Multiply and simplify $\frac{11ab^3}{5a^2b^4} \times \frac{10a^3b}{22ab}$, $a \neq 0,\ b \neq 0$.

$$= \frac{(11\times10)\times(a\times a^3)\times(b^3\times b)}{(5\times22)\times(a^2\times a)\times(b^4\times b)}$$

$$= \frac{110\ a^4b^4}{110\ a^3b^5}$$

$$= \frac{a^{4-3}}{b^{5-4}}$$

$$= \frac{a}{b}$$

(5) 2. Multiply and simplify $\frac{5x^2-45}{x^2-6x-7} \times \frac{x^2-1}{3x^2+11x+6}$.

$$= \frac{5(x^2-9)}{(x-7)(x+1)} \times \frac{(x-1)(x+1)}{(x+3)(3x+2)}$$

$$= \frac{5\overset{1}{\cancel{(x+3)}}(x-3)}{(x-7)\cancel{(x+1)}} \times \frac{(x-1)\overset{1}{\cancel{(x+1)}}}{\underset{1}{\cancel{(x+3)}}(3x+2)}$$

$$= \frac{5(x-3)(x-1)}{(x-7)(3x+2)}$$

(3) 3. Divide and simplify $\frac{25x^4y^7}{49xy^6} \div \frac{5xy^3}{14x^4y}$.

$$= \frac{\overset{5}{\cancel{25}}x^4y^7}{\underset{7}{\cancel{49}}xy^6} \times \frac{\overset{2}{\cancel{14}}x^4y}{\underset{1}{\cancel{5}}xy^3}$$

$$= \frac{10x^6}{7y}$$

(5) 4. Divide and simplify and state the nonpermissible values.

$$\frac{30x^2+x-1}{2x^2-15x-8} \div \left(5x^2-4x-1\right)$$

$$= \frac{30x^2+x-1}{2x^2-15x-8} \times \frac{1}{\left(5x^2-4x-1\right)}$$

$$= \frac{(6x-1)\cancel{(5x+1)}}{(x-8)(2x+1)} \times \frac{1}{\underset{1}{\cancel{(5x+1)}}(x-1)}$$

$$= \frac{(6x-1)}{(x-8)(2x+1)(x-1)}$$

The nonpermissible values for x are 8, $-\frac{1}{2}$, $-\frac{1}{5}$, and 1.

(5) 5. Simplify and state the nonpermissible values.

$$\frac{6x^2+13x-15}{25x^2-1} \times \frac{20x^2-x-1}{6x-5} \div \frac{4x^2+11x-3}{3}$$

$$= \frac{\overset{1}{\cancel{(6x-5)}}\overset{1}{\cancel{(x+3)}}}{\underset{1}{\cancel{(5x+1)}}(5x-1)} \times \frac{\overset{1}{\cancel{(4x-1)}}\overset{1}{\cancel{(5x+1)}}}{\underset{1}{\cancel{(6x-5)}}} \times \frac{3}{\underset{1}{\cancel{(4x-1)}}\underset{1}{\cancel{(x+3)}}}$$

$$= \frac{3}{5x-1}$$

The nonpermissible values for x are $\frac{1}{5}$, $-\frac{1}{5}$, $\frac{5}{6}$, $\frac{1}{4}$, and -3.

Topic 3

_____ marks

Topic 4: Adding and Subtracting

(7) 1. Add and simplify $\frac{x}{x+7}+\frac{3}{x^2+5x-14}$.

$$\boxed{x^2+5x-14=(x+7)(x-2)}$$

$$\text{L.C.D.}=(x+7)(x-2) \text{ or } x^2+5x-14$$

$$=\frac{x(x-2)+3}{x^2+5x-14}$$

$$=\frac{x^2-2x+3}{x^2+5x-14}$$

(9) 2. Subtract and simplify $\frac{5}{(y-5)}-\frac{y-3}{2y^2-5y-25}$.

$$2y^2-5y+25=(y-5)(2y+5)$$

$$\therefore \text{ L.C.D.}=(y-5)(2y+5) \text{ or } 2y^2-5y-25$$

$$=\frac{5(2y+5)}{(y-5)(2y+5)}-\frac{y-3}{(y-5)(2y+5)}$$

$$=\frac{10y+25-y+3}{(y-5)(2y+5)}$$

$$=\frac{9y+28}{(y-5)(2y+5)}$$

(9) 3. Simplify $\frac{x-2}{7x^2+20x-3}+\frac{3}{7x^2-8x+1}-\frac{x-1}{7x^2+13x-2}$.

L.C.D. $=(7x-1)(x+3)(x-1)(x+2)$

$$=\frac{x-2}{(7x-1)(x+3)}+\frac{3}{(7x-1)(x-1)}-\frac{x-1}{(7x-1)(x+2)}$$

$$=\frac{(x-2)(x-1)(x+2)+3(x+3)(x+2)-(x-1)(x-1)(x+3)}{(7x-1)(x+3)(x-1)(x+2)}$$

$$=\frac{x^3-4x-x^2+4+3x^2+15x+18-x^3-x^2+5x-3}{(7x-1)(x+3)(x-1)(x+2)}$$

$$=\frac{x^2+16x+19}{(7x-1)(x+3)(x-1)(x+2)}$$

Topic 4

_____ marks

Topic 5: Solving Rational Equations

(5) 1. Solve $\frac{x-1}{8x^2-11x+3}+\frac{1}{7x^2-8x+1}=\frac{1}{8x-3}$.

State the nonpermissible values for x.

$$\frac{x-1}{(8x-3)(x-1)}+\frac{1}{(x-1)(7x-1)}=\frac{1}{(8x-3)}$$

The nonpermissible values for x are 1, $\frac{3}{8}$, and $\frac{1}{7}$.

L.C.D. $=(8x-3)(x-1)(7x-1)$

$$\begin{aligned}(x-1)(7x-1)+(8x-3)&=(7x-1)(x-1)\\ 7x^2-8x+1+8x-3&=7x^2-8x+1\\ 7x^2-2&=7x^2-8x+1\\ -2&=-8x+1\\ 8x&=2+1\\ 8x&=3\\ x&=\frac{3}{8}\end{aligned}$$

The value of $x=\frac{3}{8}$ is one of the nonpermissible values; therefore, the equation does not have a solution.

(7) 2. Solve $\frac{6x^2+7x}{2x-1}-3=\frac{5}{2x-1}+x$ and check the solution.

State the nonpermissible values for x.

$\text{L.C.D.}=2x-1 \qquad x\neq\frac{1}{2}$

$$\begin{aligned}
6x^2+7x-3(2x-1)&=5+x(2x-1)\\
6x^2+7x-6x+3&=5+2x^2-x\\
6x^2+x+3&=2x^2-x+5\\
4x^2+2x-2&=0\\
2\left(2x^2+x-1\right)&=0\\
2(2x-1)(x+1)&=0\\
2x-1&=0\\
x&=\frac{1}{2}\\
x+1&=0\\
x&=-1
\end{aligned}$$

Since $x\neq\frac{1}{2}$ (nonpermissible value), $x=\frac{1}{2}$ is not acceptable.

Check: Let $x=-1$.

LS	RS
$\frac{6x^2+7x}{2x-1}-3$	$\frac{5}{2x-1}+x$
$=\frac{6(-1)^2+7(-1)}{2(-1)-1}-3$	$=\frac{5}{2(-1)-1}+(-1)$
$=\frac{6-7}{-3}-3$	$=\frac{5}{-3}-1$
$=\frac{1}{3}-3$	$=-\frac{8}{3}$
$=-2\frac{2}{3}$	$=-2\frac{2}{3}$

LS = RS

Therefore, the solution is -1.

③ 3. The denominator of a rational expression is the square of a number, less 81. The numerator is 3. If this expression is equivalent to the reciprocal of the sum of the number and 9, determine the possible values of the number.

Step 1: Let x be the number.

Step 2: $\frac{3}{x^2-81}=\frac{1}{x+9} \quad x \neq \pm 9$

Step 3: $x^2-81=3x+27$

$x^2-3x-108=0$

$(x-12)(x+9)=0$

$x-12=0 \quad x=12$

$x+9=0 \quad x=-9$

The number -9 is a nonpermissible value.

Therefore, the solution is 12.

(5) 4. The area of a triangle is $(5x^2+2x-3)$ square units. The height of the triangle is given by the expression $(5x-3)$ units. The lengths of the other two sides (not the base) are $\left(\frac{x-1}{2}\right)$ units and 3 units. If the perimeter is $\left(\frac{11}{x+2}\right)$ units, find the length of the base. First find an expression for the triangle by using the area of a triangle formula. Then set up an equation using the perimeter of a triangle formula. Solve this formula for x and use the x-values to find the exact measure for the length of the base.

Step 1: Area of a triangle $A=\frac{1}{2}bh$

$$\therefore b=\frac{2A}{h}$$

$$b=\frac{2(5x^2+2x-3)}{(5x-3)}=\frac{2\cancel{(5x-3)}^{1}(x+1)}{\cancel{(5x-3)}_{1}}=2(x+1)$$

Length of the base = perimeter – lengths of the other two sides.

Step 2: $2(x+1)=\frac{11}{x+2}-\frac{x-1}{2}-3$

Step 3:

$$2(x+1)=\frac{11}{x+2}-\frac{x-1}{2}-3$$

$$2(x+1)+\frac{x-1}{2}+3=\frac{11}{x+2}$$ (Multiply through brackets and multiply each term by 2.)

$$4x+4+x-1+6=\frac{22}{x+2}$$

$$5x+9=\frac{22}{x+2}$$ (Multiply each side by $x+2$.)

$$(5x+9)(x+2)=22$$

$$5x^2+19x+18=22$$

$$5x^2+19x-4=0$$

$$(x+4)(5x-1)=0$$

$(x+4)=0$ $\qquad$ $(5x-1)=0$

$x=-4$ $\qquad$ $5x=1$

$$x=\frac{1}{5}$$

(Continued on next page)

Length of the base $= 2(x+1)$

If $x = -4$, $2(x+1) = -6$. (The negative solution is not acceptable because length cannot be negative.)

If $x = \frac{1}{5}$, $2\left(\frac{1}{5}+1\right) = 2 \times \frac{6}{5}$

$= \frac{12}{5}$

$= 2\frac{2}{5}$ units.

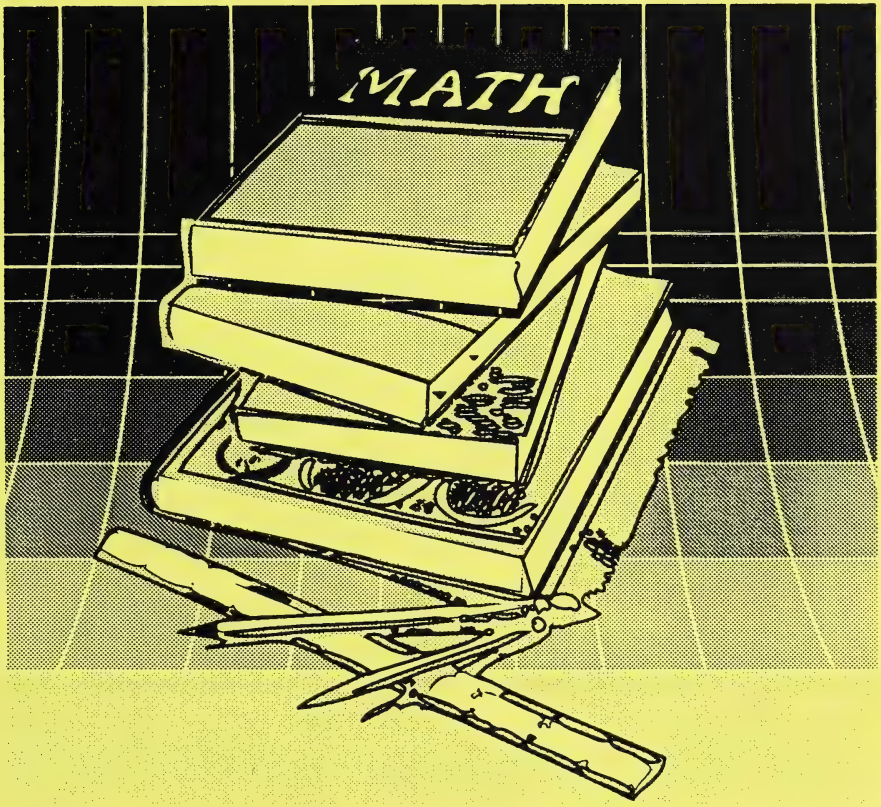
Step 4: The length of the base is $2\frac{2}{5}$ units.

Topic 5

_____ marks

UNIT 3

MATHEMATICS 20



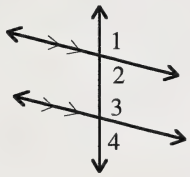
LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

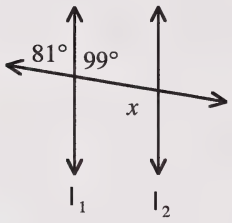
Topic 1: Parallel Lines and Related Terms

(10) 1. Accept the postulate that corresponding angles are equal. In the proof that same-side interior angles are supplementary, it is to be shown that in the diagram $\angle 2 + \angle 3 = 180°$. Fill in the missing reasons in the table below.



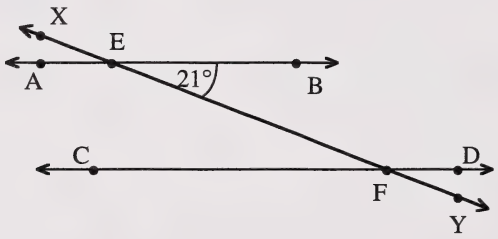
Statement	Reason
1. $\angle 1 = \angle 3$	1. **Corresponding angles are equal.**
2. $\angle 2 + \angle 1 = 180°$	2. **If two angles form a linear pair, then they are supplementary.**
3. $\angle 2 + \angle 3 = 180°$	3. comparing statements 1 and 2

(5) 2. Use the diagram to determine the value for x. $l_1 \| l_2$



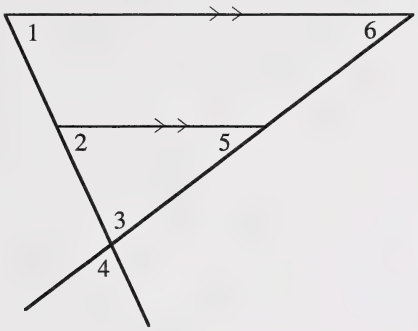
$x = 99°$ because alternate angles are equal.

(5) 3. Line segments $\overline{AB}$ and $\overline{CD}$ are parallel. Transversal $\overleftrightarrow{XY}$ cuts $\overline{AB}$ at E and $\overline{CD}$ at F. If $\angle BEF = 21°$, determine the measure of $\angle DFE$. (Draw a diagram.)



$\angle BEF$ and $\angle DFE$ are same-side interior angles and are supplementary.
$\angle DFE = 180° - \angle BEF$
$= 180° - 21° = 159°$

(5) 4. Use the diagram. $\angle 1 = 65°$ and $\angle 4 = 78°$. Determine the measure of $\angle 5$.



$\angle 2 = \angle 1 = 65°$ since corresponding angles are equal.
$\angle 3 = \angle 4 = 78°$ since vertical angles are equal.
$\angle 5 = 180° - \angle 2 - \angle 3 = 180° - 65° - 78° = 37°$

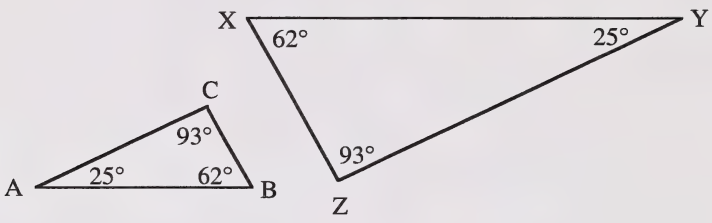
Topic 1

_____ marks

Topic 2: Similar and Congruent Triangles

(4)

1. Refer to the diagram. Suppose you were going to use Δ ABC and Δ XYZ to demonstrate that the ratios between corresponding sides of similar triangles are equal. You would show that the ratio $\frac{AB}{XY}$ is equal to which two other side ratios?



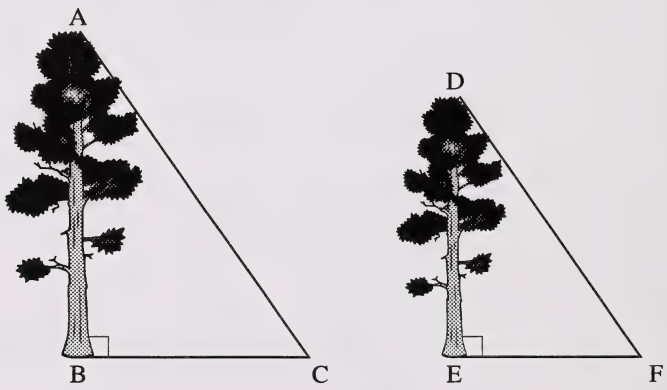
$$\frac{AB}{XY}=\frac{AC}{YZ} \text{ or } \frac{AB}{XY}=\frac{BC}{XZ} \text{ since } \angle A \leftrightarrow \angle Y$$

$$\angle B \leftrightarrow \angle X$$

$$\angle C \leftrightarrow \angle Z$$

(5)

2. On a sunny day a tree 11.50 m tall casts a shadow 8.05 m long. How tall is a second tree which has a shadow 6.44 m long?



$\Delta ABC \sim \Delta DEF$

$$\frac{AB}{BC}=\frac{DE}{EF}$$

$$\frac{11.50}{8.05}=\frac{x}{6.44}$$

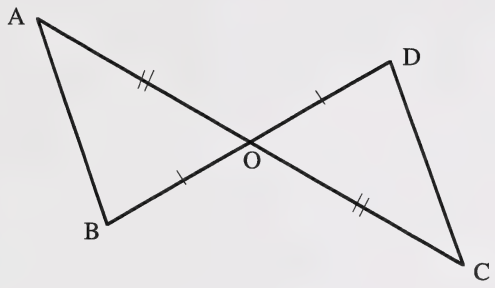
$$x=\frac{11.50\times 6.44}{8.05}$$

$$x=9.20$$

The second tree is 9.20 m high.

(4) 3. Refer to the diagram.

a. Identify two congruent triangles.

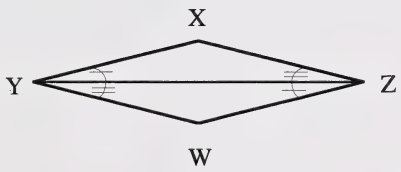


$\underline{\Delta ABO} \cong \underline{\Delta CDO}$

b. Which postulate can be used to prove the congruence of the two triangles?

SAS

(4) 4. Refer to the diagram.



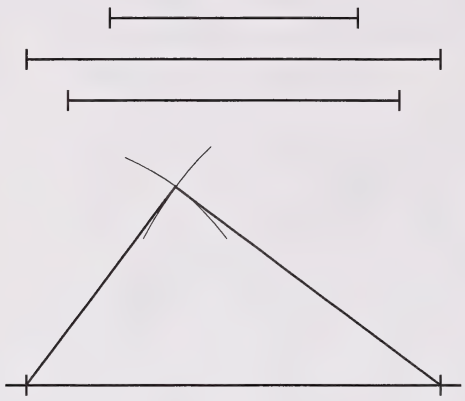
a. Identify two congruent triangles.

$\underline{\Delta XYZ} \cong \underline{\Delta WZY}$

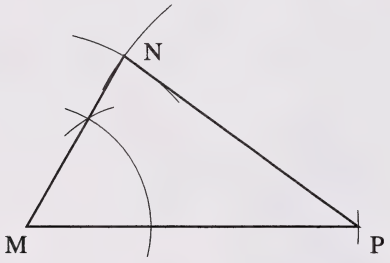
b. Which postulate can be used to prove the congruence of the two triangles?

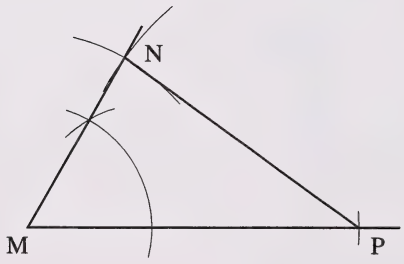
ASA

(4) 5. Refer to the diagram.
Construct by SSS a triangle with sides congruent to those given.



(4) 6. Refer to the diagram.
Construct by SAS a triangle congruent to Δ MNP.



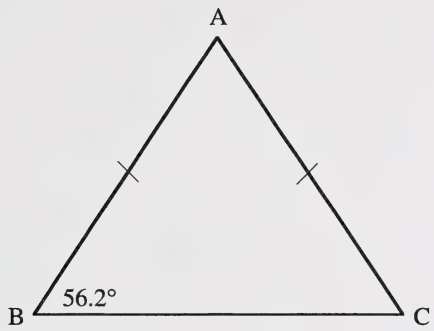


Topic 2

_____ marks

Topic 3: Triangle Geometry

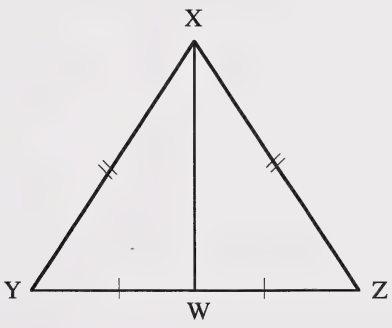
(5) 1. In Δ ABC, AB = AC and $\angle$ B = 56.2°. What is the measure of $\angle$ C? Explain your answer.



$\angle$ C = 56.2° since base angles are equal for isosceles triangles.

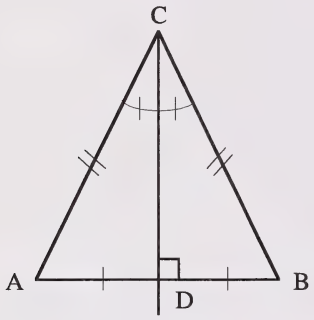
(5) 2. In proving that angles opposite the equal sides of an isosceles triangle are equal, it is shown that $\angle$ Y = $\angle$ Z in the diagram. Complete a congruency statement to tell which triangles are shown to be congruent in order to conclude that $\angle$ Y = $\angle$ Z.

$\underline{\Delta WXY} \cong \underline{\Delta WXZ}$



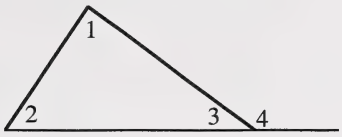
(5)

3. In the isosceles triangle ABC, AC = BC and D is the point through which the perpendicular bisector of the base cuts AB.
If $\angle ACD = 24°$, what is the measure of $\angle BCD$?
Explain how you arrived at this answer.



$\angle BCD = \angle ACD = 24°$ since the perpendicular bisector of the base bisects the vertical angle.

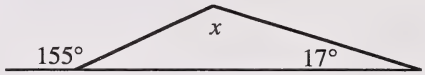
(5) 4. Refer to the diagram.
In proving that an exterior angle is equal to the sum of the remote interior angles, it is proven that $\angle 4 = \angle 1 + \angle 2$. Give the reasons for statements 1 and 2 by completing the blanks in the table below.



Statement	Reason
1. $\angle 3 + \angle 4 = 180°$	1. **$\angle 3$ and $\angle 4$ are adjacent angles making a straight line.**
2. $\angle 1 + \angle 2 + \angle 3 = 180°$	2. **The sum of the angles of a triangle is 180°.**
3. $\angle 3 + \angle 4 = \angle 1 + \angle 2 + \angle 3$	3. comparing statements 1 and 2
4. $\angle 4 = \angle 1 + \angle 2$	4. subtracting $\angle 3$ from both sides of statement 3

(5)

5. Refer to the diagram.
Determine the value of x and explain how you arrived at this answer.



$x + 17° = 155°$ by the Exterior Angle of a Triangle Theorem.
$x = 155° - 17° = 138°$

Topic 3

_____ marks

Topic 4: Circle Geometry

(5)

1. Fill in the blanks.

   a. A line segment joining two points on a circle is a **chord**.

   b. A line segment joining the centre of a circle and a point on the circle is a(n) **radius** of the circle.

   c. A portion of the circumference of a circle is called a(n) **arc**.

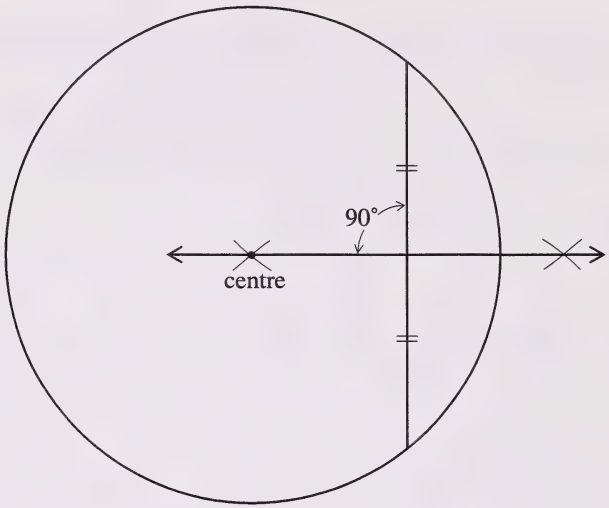
   d. A **central** angle is an angle whose vertex is on the centre of the circle.

   e. A **secant** line is a line intersecting the circle in two places.

(5) 2. Complete a diagram to verify the following statement.

A line containing the centre of a circle that bisects a chord is perpendicular to that chord.

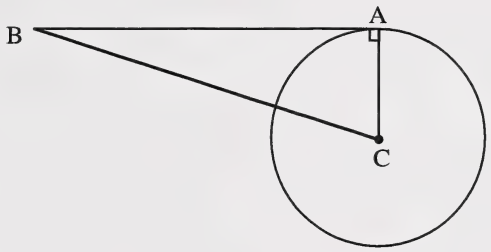
On your diagram give the angle measurement between the line and the chord.



(5)

3. Refer to the diagram.

AB = 67.0 cm and the radius of the circle has a length of 21.0 cm. $\overline{AB}$ is tangent to the circle. What is the length of $\overline{BC}$? Round your answer to one decimal place. Assume that the radius is drawn to the point of contact of the tangent. Explain how you arrived at this answer.

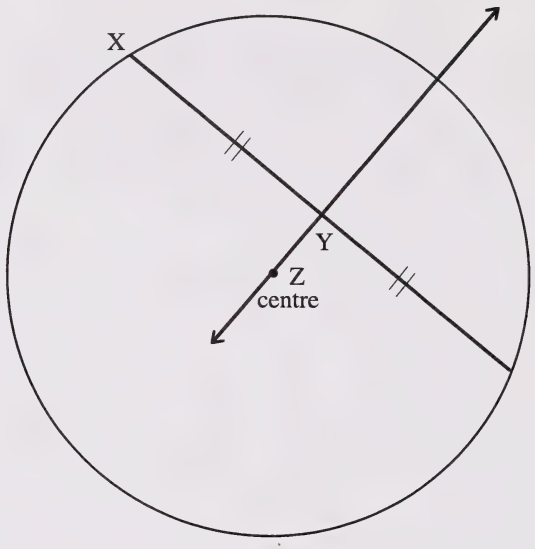


Because of the tangent-radius angle relationship, Δ ABC is a right triangle. $\overline{AC}$ is the circle radius.

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{67^2 + 21^2} = 70.2$$

The length of $\overline{BC}$ is 70.2 cm.

(5) 4. Refer to the diagram.
XY = 16.0 cm and YZ = 5.0 cm.
Determine the radius of the circle to one decimal place.
Explain how you arrived at this answer.



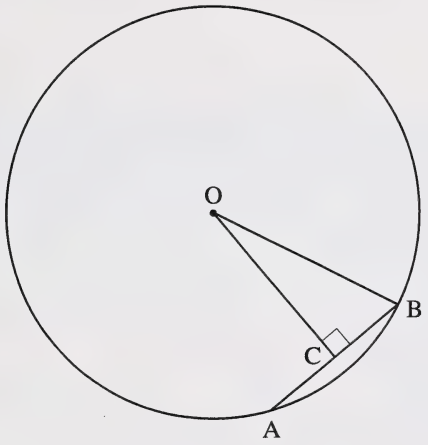
Because of the chord-bisector relationship, Δ XYZ is a right triangle. XZ is the radius.

$$XZ = \sqrt{XY^2 + YZ^2} = \sqrt{16^2 + 5^2} = 16.8$$

The radius of the circle is 16.8 cm.

(5)

5. Refer to the diagram. Chord $\overline{AB}$ is 8 cm. The length of the radius is 10 cm. Find the perpendicular distance from the centre of the circle to chord $\overline{AB}$. Round your answer to one decimal place.



$\overline{OC}^2 = \overline{OB}^2 - \overline{BC}^2$

$\overline{OC}^2 = 10^2 - 4^2$

$\overline{OC}^2 = 100 - 16$

$\overline{OC}^2 = 84$

$\overline{OC} = \sqrt{84}$

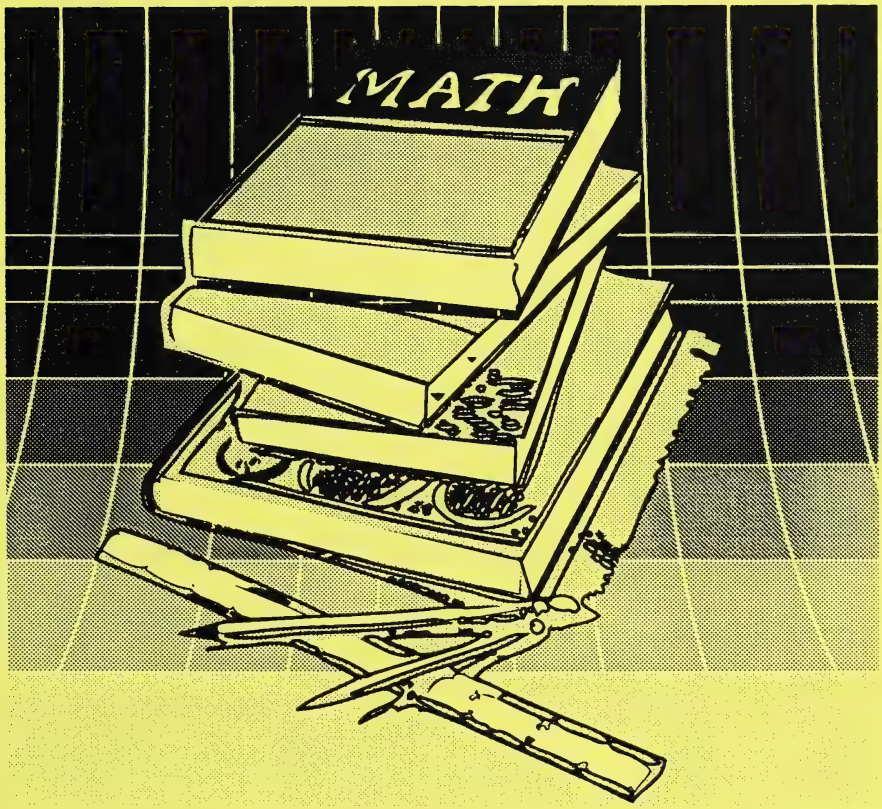
$\overline{OC} = 9.2 \text{ cm}$

Topic 4

_____ marks

UNIT 4

MATHEMATICS 20



LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

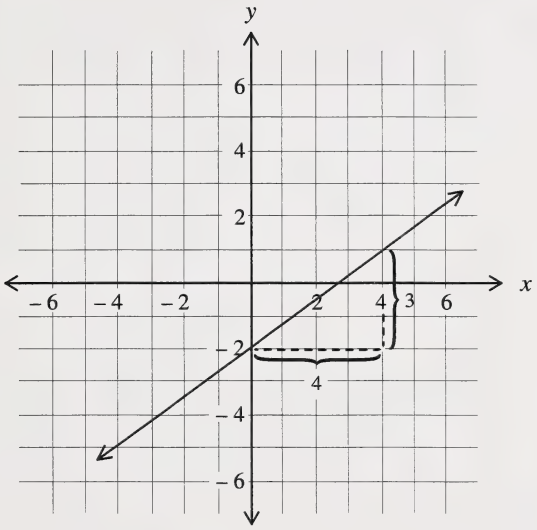
Topic 1: Relations

(4) 1. Graph $3x - 4y = 8$ by using the slope and y-intercept.

$-4y = -3x + 8$

$y = \frac{3}{4}x - 2$

Slope $= \frac{3}{4}$, y-intercept $= -2$



(3) 2. Find the domain and range of the relation $y = 3x^2$.

x can be any real number.

Domain: $D = \{x \mid x \in R\}$

x^2 is always positive.

Range: $R = \{y \mid y \geq 0,\ y \in R\}$

(4) 3. Find the domain and range of the relation $y = \frac{1}{x-5}$.

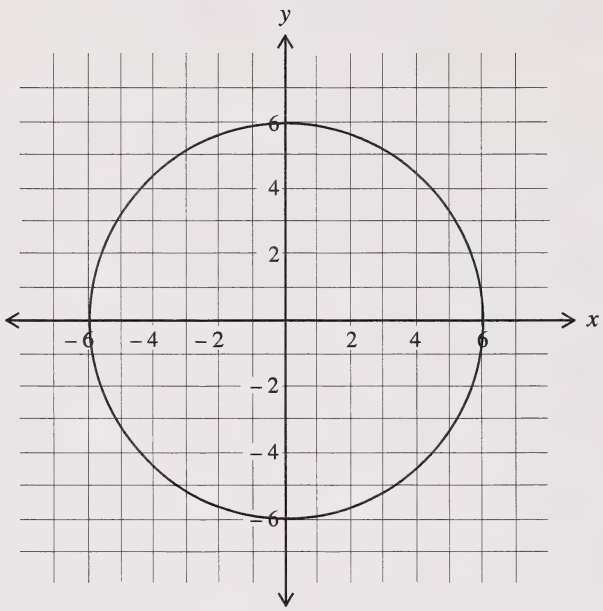
$x - 5$ cannot be zero.

Therefore, $x \neq 5$.

Domain: $D = \{x \mid x \neq 5,\ x \in R\}$

Range: $R = \{y \mid y \neq 0,\ y \in R\}$

(4) 4. State the domain and range of the following graph.



Domain: $D = \{x | -6 \le x \le 6, x \in R\}$

Range: $R = \{y | -6 \le y \le 6, y \in R\}$

Topic 1

_____ marks

Topic 2: Functions

(2) 1. Express $5x - 3y = 4$ in

a. functional notation

$$f(x) = \frac{5}{3}x - \frac{4}{3}$$

b. mapping notation

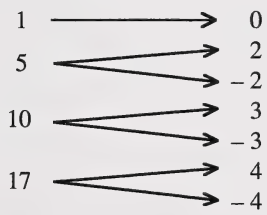
$$x \rightarrow \frac{5}{3}x - \frac{4}{3}$$

(3) 2. Draw a mapping diagram to describe the following relation; then state if it is a function.

$y^2 - x + 1 = 0$, Domain = {1, 5, 10, 17}

When $x = 1$, $y = 0$.
When $x = 5$, $y = \pm 2$.
When $x = 10$, $y = \pm 3$.
When $x = 17$, $y = \pm 4$.

Mapping diagram:



This is not a function, because there are two y-values for some x-values.

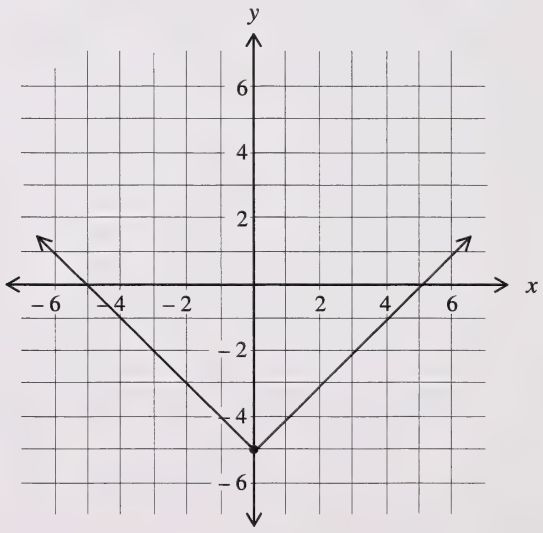
(2) 3. If $f(x)=3x-5$, find $f(x+1)$.

$$\begin{aligned} f(x+1) &= 3(x+1)-5 \\ &= 3x+3-5 \\ &= 3x-2 \end{aligned}$$

(3) 4. If $f(x)=2x+1$ and $g(x)=5x$, find $3f(x)-2g(x+1)$.

$$3f(x)=3(2x+1)$$
$$2g(x+1)=2[5(x+1)]$$
$$\begin{aligned} 3f(x)-2g(x+1) &= 3(2x+1)-10(x+1) \\ &= 6x+3-10x-10 \\ &= -4x-7 \end{aligned}$$

(5) 5. Sketch the graph of $y=|x|-5$. What are the zeros of this function?



The zeros are 5 and – 5.

(4) 6. The volume of water from a hose in a given period of time varies directly as the square of the diameter of the hose. In a certain period of time, a hose with a 3 cm diameter gives 540 kL of water. In the same period of time, how much water can a 5 cm diameter hose give?

$$V = Kd^2$$
$$540 = K(3)^2$$
$$K = \frac{540}{9}$$
$$= 60$$

$$V = Kd^2$$
$$= 60d^2$$
$$= 60(5)^2$$
$$= 1500 \text{ kL}$$

A 5 cm diameter hose can give 1500 kL of water.

(3) 7. If temperature remains constant, the volume of gas is inversely proportional to the pressure. If the volume is 600 mL at a pressure of 15 kPa (kilopascals), what is the volume at a pressure of 25 kPa?

$$V = \frac{K}{P}$$
$$\therefore K = VP$$
$$K = K \therefore VP = VP$$
$$15 \times 600 = 25 \times V$$
$$V = \frac{15 \times 600}{25}$$
$$= 360 \text{ mL}$$

The volume at a pressure of 25 kPa is 360 mL.

(10) 8. The Fast Car Rental Company rents cars for \$32 per day plus \$0.50/km driven.

a. Write an equation to represent this relation.

Let C = total cost. d = kilometres driven

$C = \$32 + \$0.50\,d$ or $C = \frac{1}{2}d + 32$

b. Find the total cost per day if 50 km, 80 km, and 90 km were driven.

If $d = 50$, then $C = \frac{1}{2} \times 50 + 32$

$= \$57.$

If $d = 80$, then $C = \frac{1}{2} \times 80 + 32$

$= \$72.$

If $d = 90$, then $C = \frac{1}{2} \times 90 + 32$

$= \$77.$

c. What is the independent variable, and what is the dependent variable?

C is the dependent variable, and d is the independent variable.

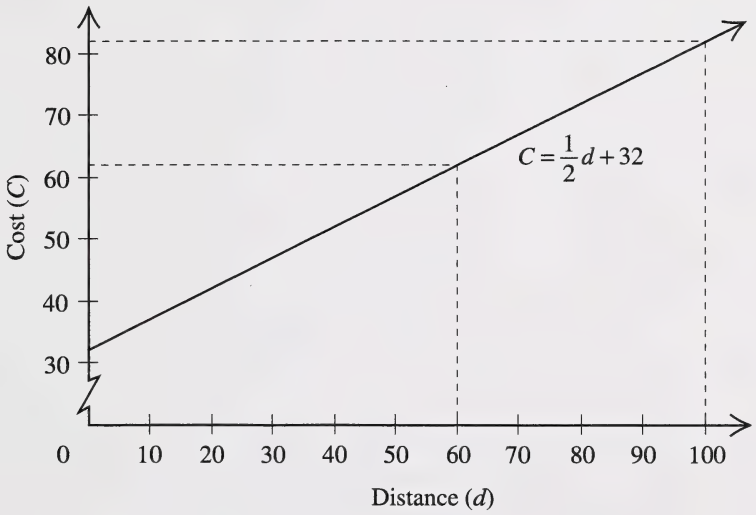
d. Determine the domain and range.

Domain: $D = \{d \mid d \geq 0,\ d \in R\}$

Range: $R = \{C \mid C \geq 32,\ C \in R\}$

e. Draw the graph.

d	50	80	90	0
C	57	72	77	32



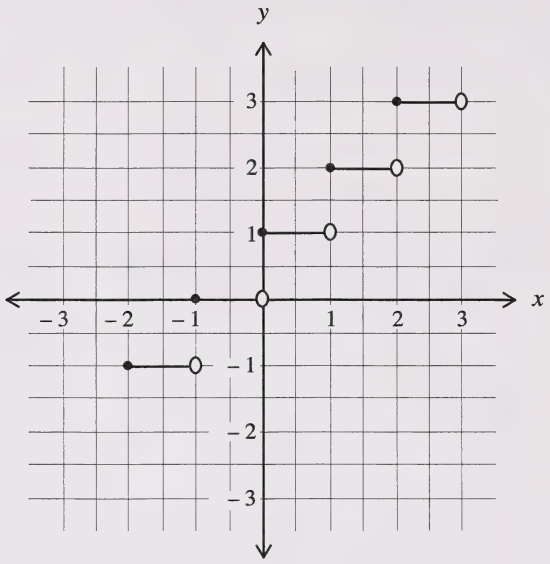
f. Interpolate from the graph the total cost per day if 60 km have been driven.

When $d = 60$ km,
$C = \$62$.

g. Extrapolate from the graph the total cost per day if the distance driven is 100 km.

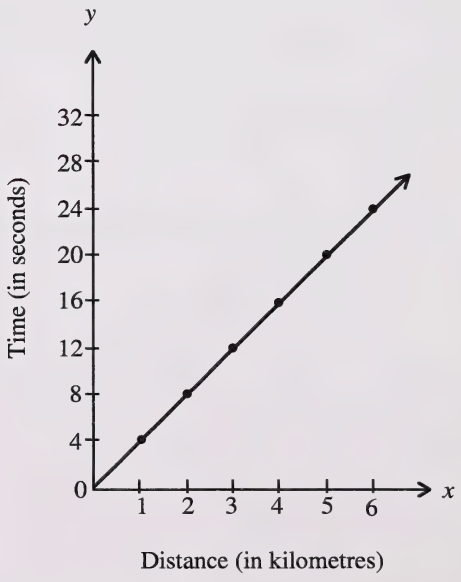
When $d = 100$ km,
$C = \$82$.

(2) 9. Sketch the graph $f(x)=[x+1]$. (Note that this is the step function.)



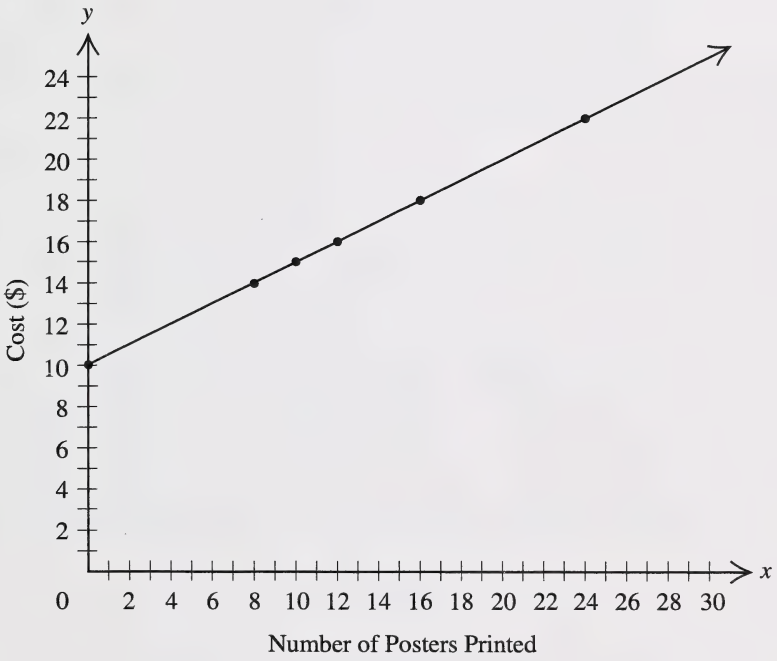
(2) 10. The time it takes you to hear the soun ' of thunder varies directly as your distance from the storm. Suppose for a storm 1 km away, it takes you 4 s to hear the thunder. How long would it take you to hear the thunder if the storm were 2 km, 3 km, 4 km, 5 km, and 6 km away? Complete the table of values and sketch the graph.

x	1	2	3	4	5	6
y	**4**	**8**	**12**	**16**	**20**	**24**



③ 11. Ace Printing charges a flat fee of $10 plus $0.50 per copy to print campaign posters. How much would it cost to print 10, 16, 24, 12, and 8 posters? Make a graph for this information.

x	0	8	10	12	16	24
y	10	14	15	16	18	22



(5) 12. A high school has 670 students. Their Valentine's dance tickets (t) sell for \$5 each. The cost of music, decorations, and lunch supplies totals \$935. Only students from this particular school can attend.

a. Write a rule in function notation that will determine the profit (P) for tickets selling at \$5 each with a cost of \$935.

$P = 5t - 935$

b. What is the range of profit available to the dance committee?

They could range between losing \$935 and making a profit of \$2415.

$R = \{-935 \text{ to } 2415\}$

c. How many students must attend so that the break-even point is reached?

To break even, the cost of the dance (\$935) must be covered, and the profit is zero.

$$P = 5t - 935$$
$$0 = 5t - 935$$
$$5t = 935$$
$$t = 187$$

187 students must attend.

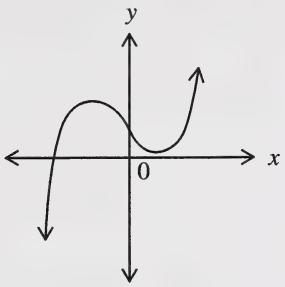
d. What is the domain of possible students attending this dance?

The domain of the students attending the dance could be between none attending and everyone attending.

$D = \{\text{integers from } 0 \text{ to } 670\}$

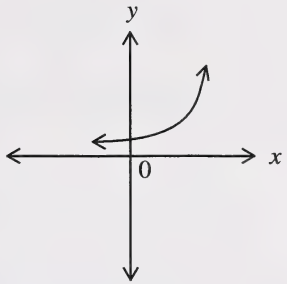
(6) 13. Name each of the following functions.

a.



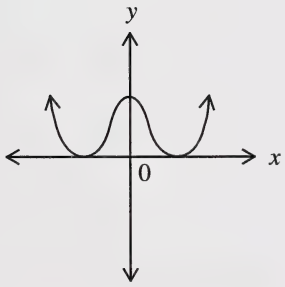
cubic function

b.



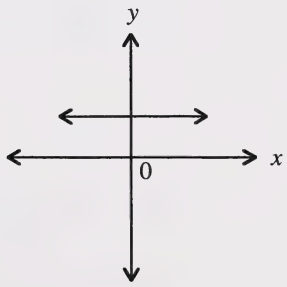
exponential function

c.



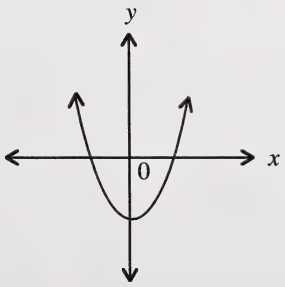
degree-4 polynomial function

d.



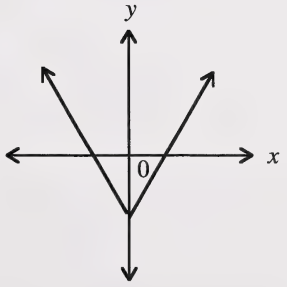
constant function

e.



quadratic function

f.



absolute-value function

(10) 14. Classify each function.

a. The value of \$100 when invested for n years at 10% compounded annually is $A = 100(1.1)^n$.

exponential function

b. A function is represented by $y = 2|x+1|-2$.

absolute-value function

c. The height, h, in metres, reached by a ball after n bounces is given by $h = 2(0.7)^n$.

exponential function

d. The number of bacteria, N, in a culture t hours after midnight is given by $N = 1000 \times 2^t$.

exponential function

e. The length, L, in metres, of a steel girder is $L = 8.5 + 0.000\,12\,(t - 20)$ where t is the temperature in °C.

linear function

f. At an interest rate, i, the amount that must be invested now to have \$100 in 1 year is $P = \frac{100}{1+i}$.

rational function

g. To deliver parcels in a metropolitan area, Rapidsend Courier Service charges \$5 for the first kilogram plus \$2 for each additional kilogram or part thereof.

linear function

h. A car begins to coast down a hill. The distance, d, in metres, it rolls in t seconds is $d = 0.6t^2$.

quadratic function

i. The value, V, in dollars, of a car is given by $V = \frac{12\ 000}{n+1} + 400$ where n is its age in years.

rational function

j. The free end of a diving board dips d cm when under a load of x kg.
The relation is $d = 0.00001x^3 + 0.0005x^2$.

cubic function

Topic 2

_____ marks

Topic 3: Inverses of Relations and Functions

(3) 1. If $2y^2 - 3y + 4x - 7 = 0$, find the inverse of this relation and determine if the inverse is a function.

Inverse: $2x^2 - 3x + 4y - 7 = 0$

$$4y = -2x^2 + 3x + 7$$

$$y = -\frac{1}{2}x^2 + \frac{3}{4}x + \frac{7}{4}$$

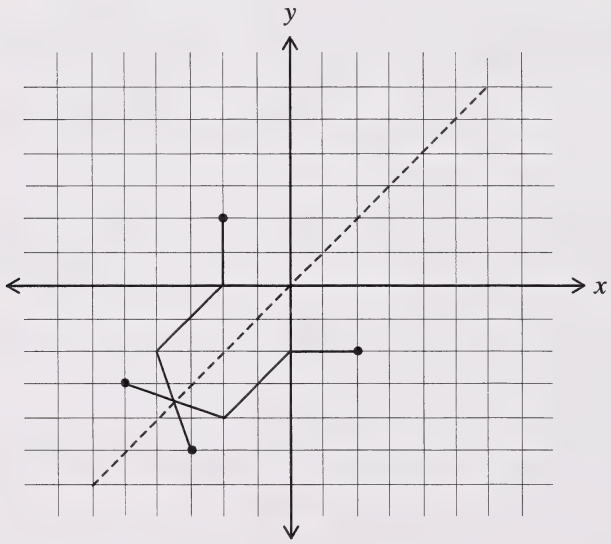
The inverse is a function.

(3) 2. Find the inverse of $T = \{(5, 2), (3, 8), (7, 2), (6, 9)\}$. Is the inverse of T a function?

$T^{-1} = \{(2, 5), (8, 3), (2, 7), (9, 6)\}$

T is a function, and T^{-1} is not a function.

(4) 3. Graph the inverse of the relation shown on the grid.



(2) 4. Find the inverse of $y = 11$. Is the inverse a function?

The inverse is $x = 11$. It is not a function.

Topic 3

_____ marks

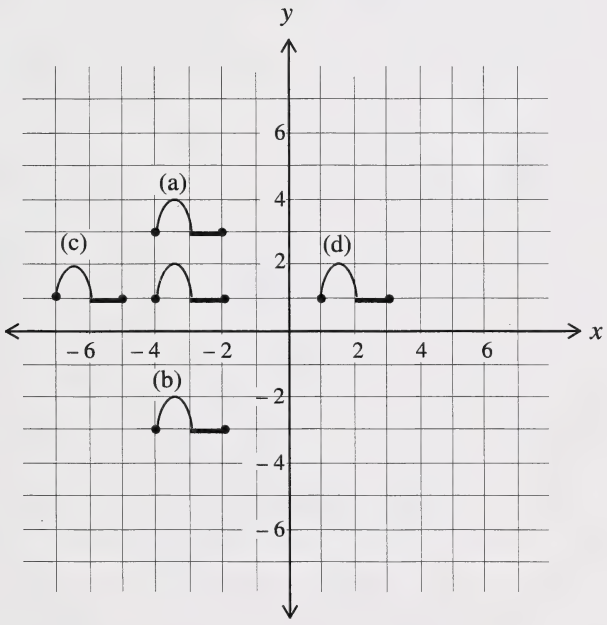
Topic 4: Transformations on Functions

(4)

1. The graph of $y = f(x)$ is shown below. On the same coordinate plane, sketch the graphs of the following:

   a. $y = f(x)+2$ b. $y = f(x)-4$

   c. $y = f(x+3)$ d. $y = f(x-5)$



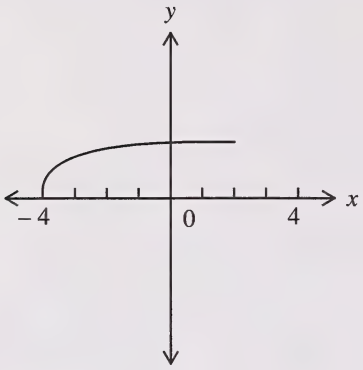
(2)

2. Use functional notation to denote the new function if the graph of $y = |x|+2$ is moved 3 units to the right and 5 units up.

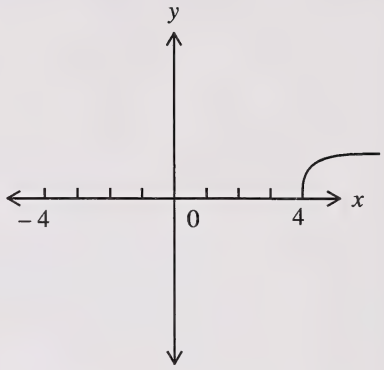
   $f(x) = |x-3|+7$

(2) 3. Which of the following graphs best represents $y = \sqrt{x-4}$?
Circle the correct letter(s).

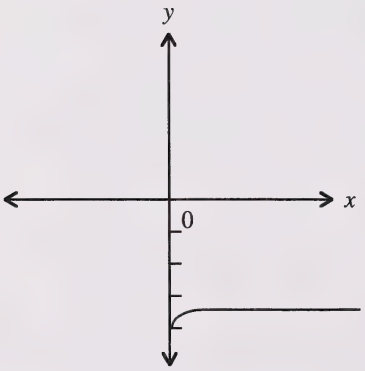
a.



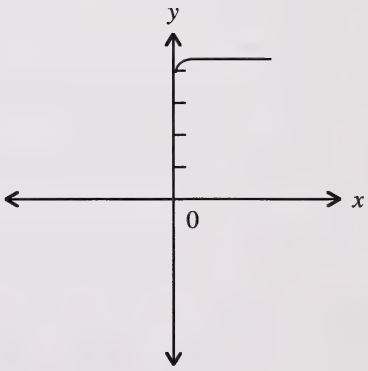
(b.)



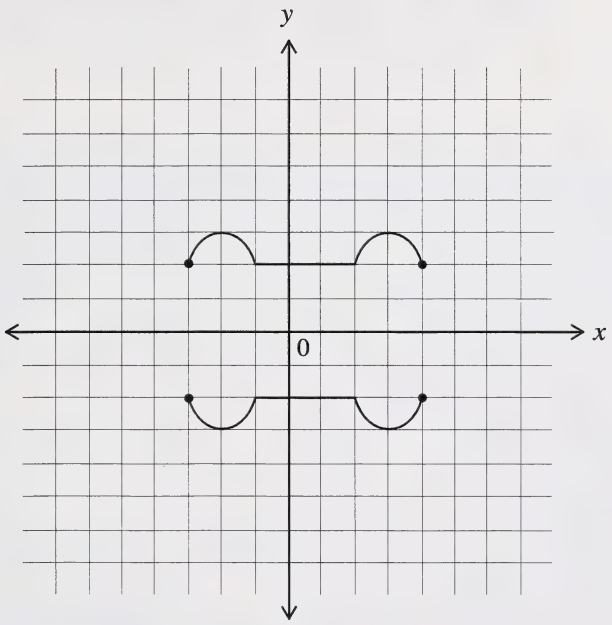
c.



d.



(2) 4. The graph of $y = f(x)$ is shown below. Sketch the graph of $y = -f(x)$.



(3) 5. Use functional notation to denote

a. the graph of $y = (x-1)^3$ stretched vertically by a factor of 5

$f(x) = 5(x-1)^3$

b. the graph of $y = (x-1)^3$ compressed vertically by the factor of $\frac{1}{5}$

$f(x) = \frac{1}{5}(x-1)^3$

c. the reflection of $y = (x-1)^3$ in the x-axis

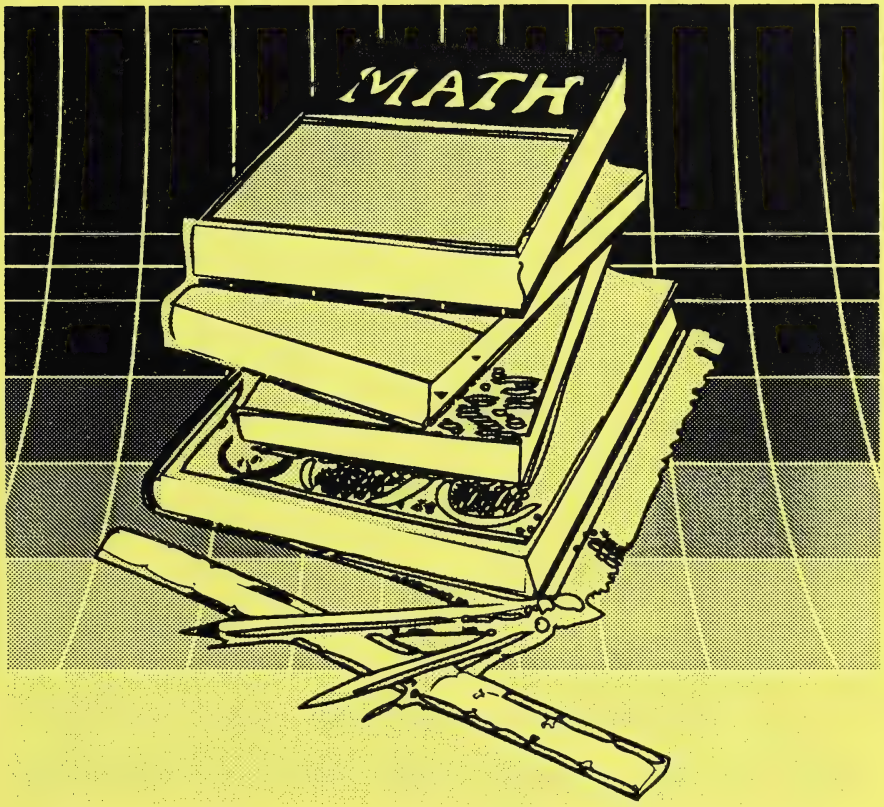
$f(x) = -(x-1)^3$

Topic 4

_____ marks

UNIT 5

MATHEMATICS 20



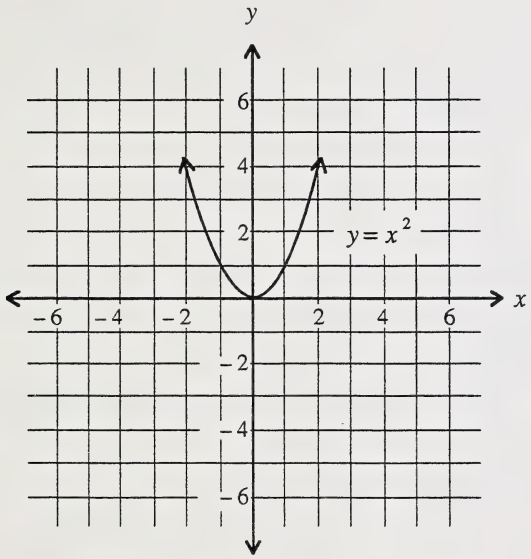
LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta
EDUCATION

Topic 1: Sketching Quadratic Functions

(1) 1. Sketch the graph of $y = x^2$.



(1) 2. How can you tell from looking at the equation of a function whether the graph of the function will be a parabola?

If the function can be written in the form $y = ax^2 + bx + c$, then the graph will be a parabola.

(1) 3. What determines whether a parabola opens upwards or downwards?

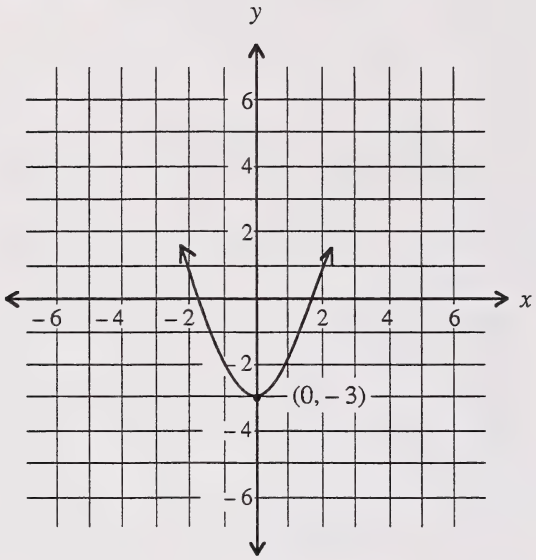
The sign of a in $y = ax^2 + bx + c$ determines the direction of opening.

(1) 4. What is the axis of symmetry of a parabola?

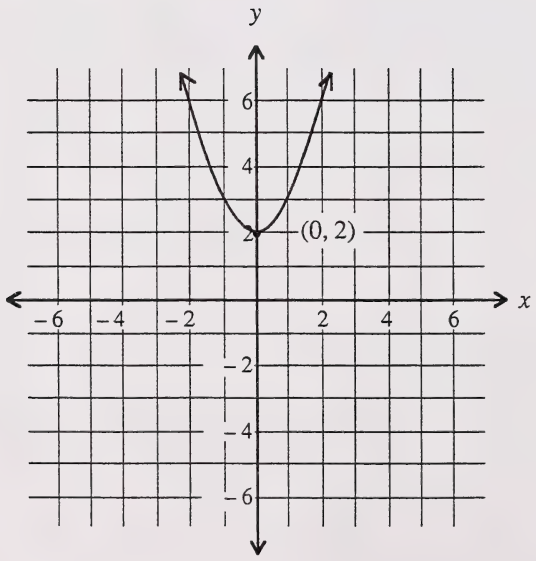
The axis of symmetry is a line that divides the parabola into two congruent parts.

④ 5. Sketch the graph of each of the following:

a. $y = x^2 - 3$



b. $y = x^2 + 2$



(2) 6. For each quadratic function below, identify the coordinates of the vertex.

a. $y = x^2 + \sqrt{2}$

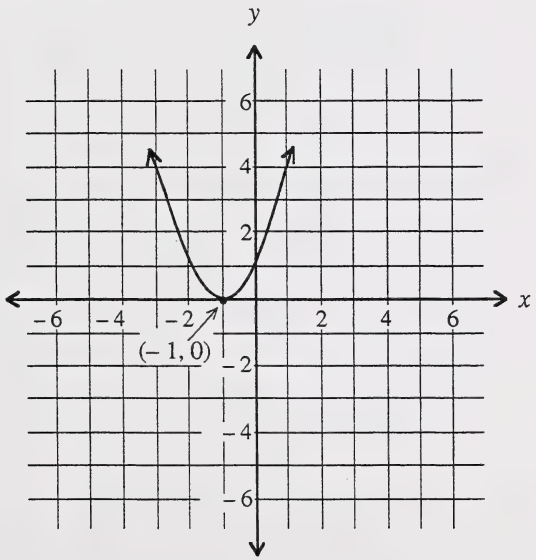
The vertex is at $\left(0, \sqrt{2}\right)$.

b. $y = x^2 - \pi$

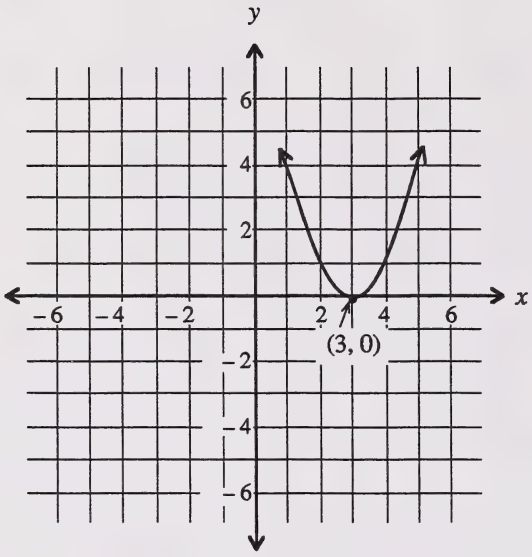
The vertex is at $(0, -\pi)$.

(4) 7. Sketch the graph of each of the following functions.

a. $y = (x+1)^2$



b. $y=(x-3)^2$



(2) 8. Identify the coordinates of the vertex for each function below.

a. $y=(x-2)^2$

The vertex is at (2, 0).

b. $y=(x+M)^2$

The vertex is at $(-M, 0)$.

(2) 9. Write the equation for the axis of symmetry for each of the following:

a. $y=x^2-5$

$x=0$

b. $y=(x+3)^2$

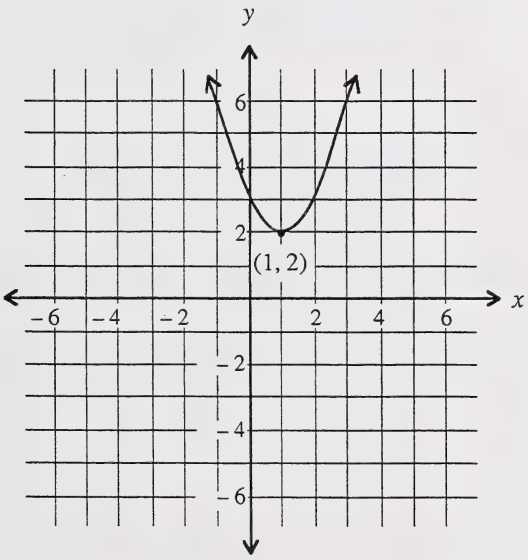
$x=-3$

(2) 10. Describe how the graph of $y=(x-3)^2+5$ can be obtained from the graph of $y=x^2$.

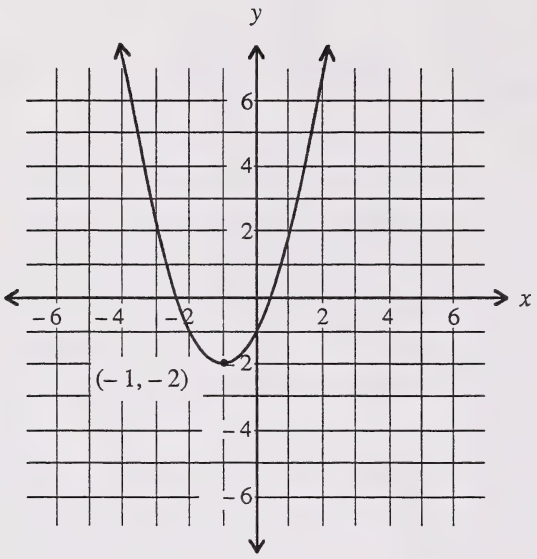
The graph of $y=(x-3)^2+5$ is congruent to the graph of $y=x^2$ translated 3 units to the right and 5 units upward.

(4) 11. Sketch the graph of each of the following functions.

a. $y=(x-1)^2+2$



b. $y=(x+1)^2-2$



(1) 12. How does the graph of $y=x^2+5$ and the graph of $y=3x^2+5$ compare?

The graph of $y=3x^2+5$ is narrower than the graph of $y=x^2+5$.

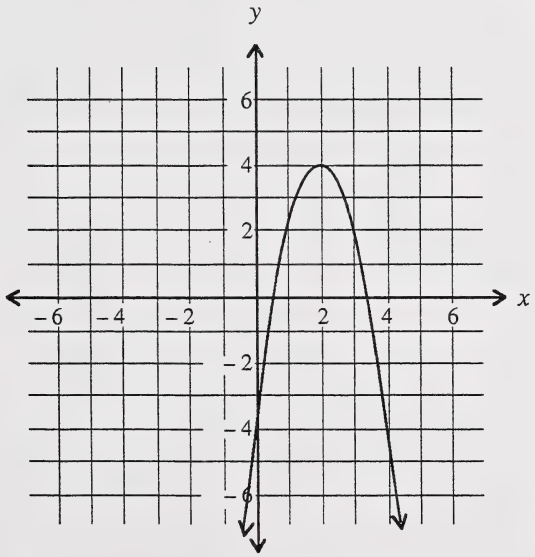
(1) 13. How does the graph of $y=-2(x-3)^2+5$ and the graph of $y=2(x-3)^2+5$ compare?

The graph of $y=-2(x-3)^2+5$ opens downward and the graph of $y=2(x-3)^2+5$ opens upward. The two graphs are congruent in shape.

(1) 14. What is the difference between the graph of $y=5x^2$ and $y=\frac{1}{5}x^2$?

The graph of $y=5x^2$ is narrower than the graph of $y=\frac{1}{5}x^2$. (The graph of $y=\frac{1}{5}x^2$ is wider than the graph of $y=5x^2$.)

(3) 15. Write the function for the parabola below.



The vertex is at (2, 4).
$a=-2$
The equation is $y=-2(x-2)^2+4$.

Topic 1

_____ marks

Topic 2: Completing The Square

(1) 1. Why is it easier to graph a quadratic function if the equation is in completed square form rather than the form $y = ax^2 + bx + c$?

You can easily identify the coordinates of the vertex if the equation is in completed square form.

(2) 2. What is the formal definition of a quadratic function?

A quadratic function is any function that can be written in the form $y = ax^2 + bx + c$ where $a, b, c \in R$, and $a \neq 0$.

(2) 3. Write the function $y - 3x^2 + 2x = 3$ in the form $y = ax^2 + bx + c$ and identify the values of a, b, and c.

$y = 3x^2 - 2x + 3$

$a = 3, \quad b = -2, \quad c = 3$

(1) 4. For $x^2 - 12x + c$, what must c be in order for the trinomial to be a perfect square?

$$c = \left(\frac{1}{2} \times -12\right)^2 = (-6)^2 = 36$$

(6) 5. Write each equation below in completed square form and identify the coordinates of the vertex.

a. $y = 2x^2 + 18$

$y = 2(x + 0)^2 + 18$

The vertex is at (0, 18).

b. $y = -4x^2 - 20x$

$y = -4\left(x^2 + 5x \quad\right)$

$y = -4\left(x^2 + 5x + \frac{25}{4}\right) - (-4)\left(\frac{25}{4}\right)$

$y = -4\left(x + \frac{5}{2}\right)^2 + 25$

The vertex is at $\left(\frac{-5}{2}, 25\right)$.

(6) 6. Write each function below in completed square form and identify the coordinates of the vertex.

a. $y = 3x^2 + 6x - 17$

$y = 3\left(x^2 + 2x \quad\right) - 17$

$y = 3\left(x^2 + 2x + 1\right) - 17 - 3$

$y = 3(x + 1)^2 - 20$

The vertex is at (– 1, – 20).

b. $y = -2x^2 + 5x + 1$

$y = -2\left(x^2 - \frac{5}{2}x \quad\right) + 1$

$y = -2\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) + 1 - (-2)\left(\frac{25}{16}\right)$

$y = -2\left(x - \frac{5}{4}\right)^2 + 1 + \frac{25}{8}$

$y = -2\left(x - \frac{5}{4}\right)^2 + \frac{33}{8}$

The vertex is at $\left(\frac{5}{4}, \frac{33}{8}\right)$.

(2) 7. Write the function $y = ax^2 + bx + c$ in completed square form.

$$y = ax^2 + bx + c$$

$$y = a\left(x^2 + \frac{b}{a}x \quad\right) + c$$

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right)$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac}{4a} - \frac{b^2}{4a}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

Topic 2

_____ marks

Topic 3: Characteristics of Quadratic Functions

(2) 1. How do you determine the maximum or minimum value of a quadratic funtion?

To find the maximum or minimum value, you find the coordinates of the vertex. The maximum or minimum value is the value of the second coordinate of the vertex.

(2) 2. How can you tell if a quadratic function has a minimum or maximum by using one of the following methods?

a. looking at the equation of the function

If $a > 0$, then a minimum value occurs. If $a < 0$, then a maximum value occurs.

b. looking at the graph of the function

If the parabola opens upward, then a minimum value occurs. If the parabola opens downward, then a maximum value occurs.

(1) 3. For $y = ax^2 + bx + c$, why is y called the dependent variable?

The value of y is dependent upon the replacement that is made for x.

(16) 4. For the quadratic function $y = -(x+2)^2 - 1$, give each of the following:

a. the coordinates of the vertex

$(-2, -1)$

b. the maximum or minimum value

maximum value of -1

c. the equation for the axis of symmetry

$x = -2$

d. the domain

$x \in R$

e. the range

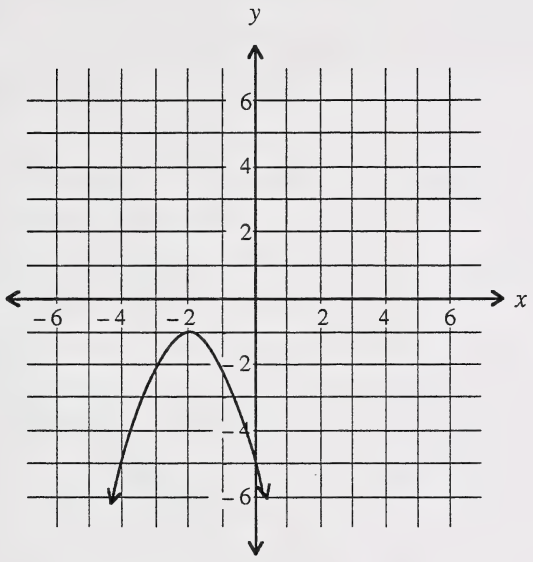
$y \leq -1$

f. the y-intercept

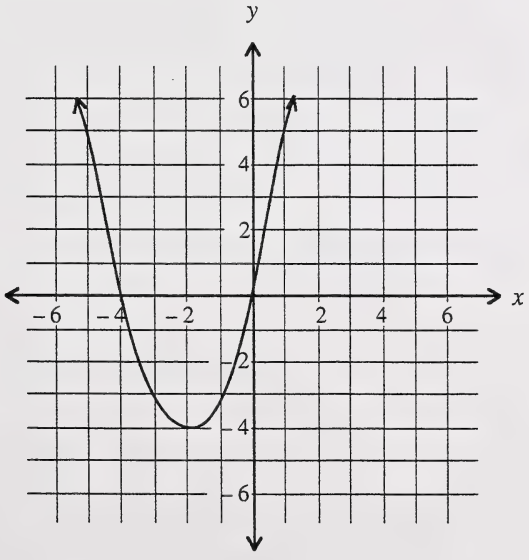
Let $x = 0$.

$y = -(0+2)^2 - 1$
$y = -(2)^2 - 1$
$y = -4 - 1$
$y = -5$

g. a sketch of the graph



(16) 5. The following quadratic function has been given in graph form.



For the quadratic function given on the previous page, determine the following:

a. the coordinates of the vertex

$(-2, -4)$

b. the maximum or minimum value

minimum value of -4

c. the equation for the axis of symmetry

$x = -2$

d. the domain

$x \in R$

e. the range

$y \geq -4$

f. the y-intercept

The y-intercept is 0.

g. the equation of the graph

$y = (x+2)^2 - 4$

(3) 6. The trajectory of a projectile is described by the function $h = -5t^2 + 140t$ where h is the height above the ground in metres and t is the time in seconds.

a. Find the time required for the projectile to reach maximum height.

$h = -5(t^2 - 28t \quad)$

$h = -5(t^2 - 28t + 196) - (-5)(196)$

$h = -5(t-14)^2 + 980$

The time required to reach maximum height is 14 s.

b. Find the maximum height reached.

The maximum height reached is 980 m.

(4) 7. Find two numbers whose sum is K and whose product is a maximum. What is the maximum product?

Let n be one of the numbers.
$K - n$ must, therefore, be the other number.

Since the product is to be a maximum then
$P = n(K - n)$ is the function to use.

$P = nK - n^2$

$P = -n^2 + nK$ (Now complete the square.)

$P = -(n^2 - nK \quad)$

$P = -\left(n^2 - nK + \frac{K^2}{4}\right) - \left(-\frac{K^2}{4}\right)$

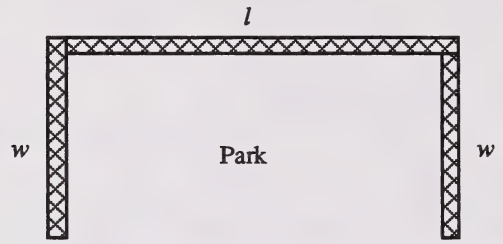
$P = -\left(n - \frac{K}{2}\right)^2 + \frac{K^2}{4}$

a. One of the numbers is $\frac{K}{2}$. The other number is $\left(K - \frac{K}{2}\right) = \frac{K}{2}$. The two numbers are $\frac{K}{2}$ and $\frac{K}{2}$.

b. The maximum product is $\frac{K^2}{4}$.

④ 8. 1000 m of wire are available for fencing three sides of a rectangular community park.

a. What should the dimensions of this park be in order to obtain a maximum area?



Let the width of the park be represented by w. The length must, therefore, be equal to $1000 - 2w$.
Since area is equal to length × width,

$(2w + l = 1000$
$l = 1000 - 2w)$

$A = (1000 - 2w)w.$

$A = 1000w - 2w^2$

$A = -2w^2 + 1000w$ (Complete the square.)

$A = -2\left(w^2 - 500w \quad \right)$

$A = -2\left(w^2 - 500w + 62\ 500\right) - (-2)(62\ 500)$

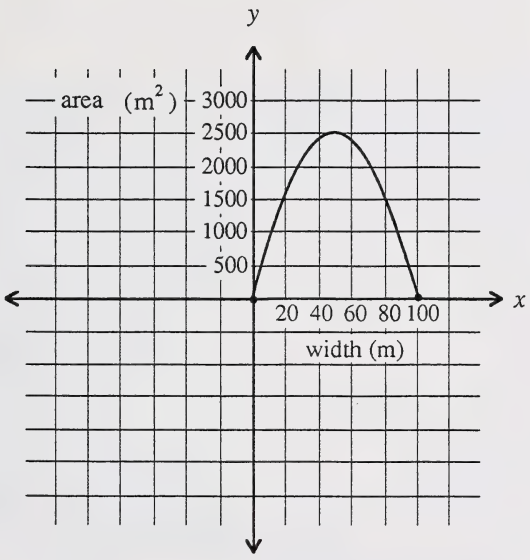
$A = -2(w - 250)^2 + 125\ 000$

The width of the park is 250 m.
The length of the park is $= 1000 - 2(250)$
$= 1000 - 500$
$= 500$ m.

b. What is this maximum area?

The maximum area is 125 000 m^2.

(2) 9. The graph below shows the relationship between the width of a rectangular region and its area. Read this graph and identify the width that will give maximum area. Also identify the maximum area.



The vertex is at (50, 2500).

The width that gives maximum area is 50 m.

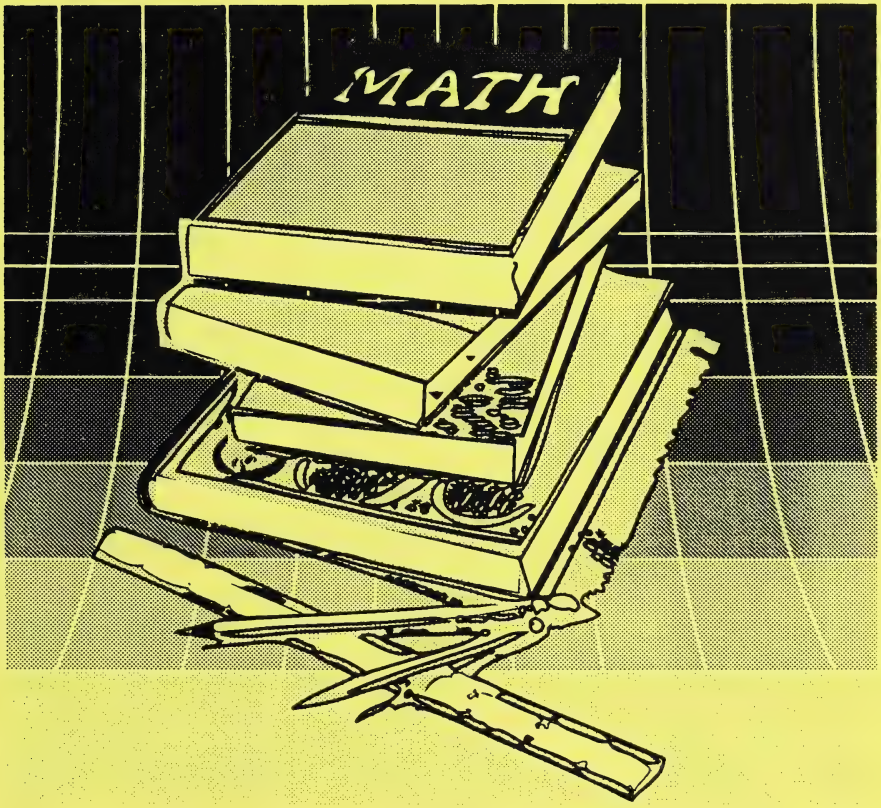
The maximum area is 2500 m^2.

Topic 3

_____ marks

UNIT 6

MATHEMATICS 20



LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

Topic 1: Computing Roots of a Quadratic Equation

(2) 1. Identify whether each of the following is a quadratic equation. Circle either yes or no.

a. $6-4x=3x^2$ (Yes) No

b. $3x^3+x-4x^2=0$ Yes (No)

c. $4xy-x+7=0$ Yes (No)

d. $-11=-4x+3x^2$ (Yes) No

(6) 2. Find the roots for each of the following quadratic equations. Use the square root method.

a. $9x^2-100=0$

$$9x^2=100$$
$$x^2=\frac{100}{9}$$
$$\sqrt{x^2}=\pm\sqrt{\frac{100}{9}}$$
$$x=\pm\frac{10}{3}$$

The possible roots are $\frac{10}{3}$ and $\frac{-10}{3}$.

b. $a^2-5=0$

$$a^2=5$$
$$\sqrt{a^2}=\pm\sqrt{5}$$
$$a=\pm\sqrt{5}$$

The possible roots are $\sqrt{5}$ and $-\sqrt{5}$.

c. $y^2+25=0$

$$y^2=-25$$
$$\sqrt{y^2}=\pm\sqrt{-25}$$
$$y=\pm\sqrt{-25}$$

There are no real number roots.

3. Find the roots for each of the following quadratic equations. Use the factoring method. Check your answers.

(4)

a. $x^2+3x+2=0$

$x^2+3x+2=0$

$(x+2)(x+1)=0$

$x+2=0$ or $x+1=0$

$x=-2$ $\quad x=-1$

The possible roots are -2 and -1.

Check for $x=-2$.

LS	RS
x^2+3x+2	0
$=(-2)^2+3(-2)+2$	
$=4-6+2$	
$=-2+2$	
$=0$	
LS = RS	

Check for $x=-1$.

LS	RS
x^2+3x+2	0
$=(-1)^2+3(-1)+2$	
$=1-3+2$	
$=-2+2$	
$=0$	
LS = RS	

The roots are -2 and -1.

④ b. $2a^2 - 2a + 6 = a^2 + 3a$

$$a^2 - 5a + 6 = 0$$
$$(a-2)(a-3) = 0$$
$$a - 2 = 0 \text{ or } a - 3 = 0$$
$$a = 2 \qquad a = 3$$

The possible roots are 2 and 3.

Check for $a = 2$.

LS	RS
$2a^2 - 2a + 6$	$a^2 + 3a$
$= 2(2)^2 - 2(2) + 6$	$= (2)^2 + 3(2)$
$= 2(4) - 4 + 6$	$= 4 + 6$
$= 8 - 4 + 6$	$= 10$
$= 4 + 6$	
$= 10$	

LS = RS

Check for $a = 3$.

LS	RS
$2a^2 - 2a + 6$	$a^2 + 3a$
$= 2(3)^2 - 2(3) + 6$	$= (3)^2 + 3(3)$
$= 2(9) - 6 + 6$	$= 9 + 9$
$= 18 - 6 + 6$	$= 18$
$= 12 + 6$	
$= 18$	

LS = RS

The roots are 2 and 3.

Topic 1

_____ marks

Topic 2: Relationship Between Equations and Functions

(2) 1. Give the corresponding function for each of the following quadratic equations.

a. $6x^2 - 5x - 7 = 0$

$y = 6x^2 - 5x - 7$

b. $-7 = 6x - 9x^2$

$9x^2 - 6x - 7 = 0$

$y = 9x^2 - 6x - 7$

(6) 2. Give the corresponding function for the quadratic equation $m^2 - 8m + 12 = 0$. Find the roots of the equation. Graph the function on the next page. Compare the roots of the equation and the x-intercepts of the function.

Equation

$$m^2 - 8m + 12 = 0$$
$$(m-6)(m-2) = 0$$
$$m - 6 = 0 \text{ or } m - 2 = 0$$
$$m = 6 \qquad m = 2$$

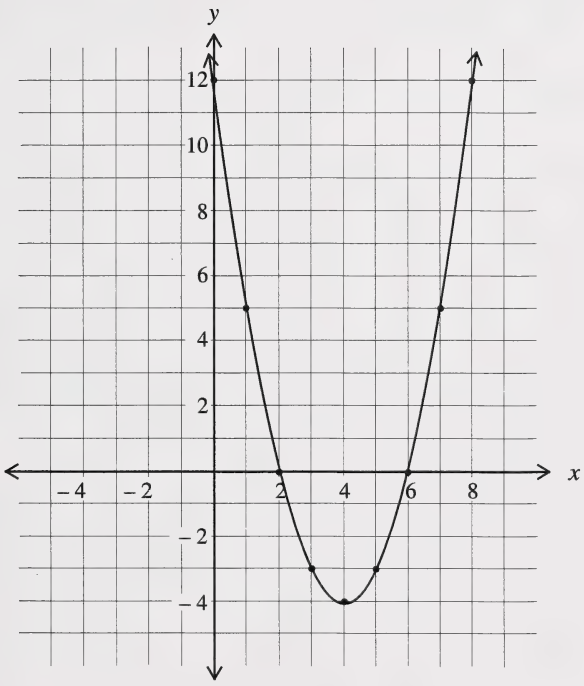
The possible roots are 6 and 2.

The graph for this question must be drawn on the following page.

Function

$y = m^2 - 8m + 12$

x	0	1	2	3	4	5	6	7	8
y	12	5	0	-3	-4	-3	0	5	12



The x-intercepts for $y = m^2 - 8m + 12$ are 6 and 2 or $(6, 0)$ and $(2, 0)$.
This shows that the roots of the equation and the x-intercepts have the same value.

Topic 2

_____ marks

Topic 3: General Form of a Quadratic Equation

(6)

1. Give the general or standard form for each of the following quadratic equations and specify the values for a, b, and c.

a. $-7x = -6 - 2x^2$

$2x^2 - 7x + 6 = 0$

$a = 2,\ b = -7,$ and $c = 6$

b. $4m^2 = -12 + 9m$

$4m^2 - 9m + 12 = 0$

$a = 4,\ b = -9,$ and $c = 12$

c. $9n(n - 2n - 3) + 1 = 0$

$9n^2 - 18n^2 - 27n + 1 = 0$

$-9n^2 - 27n + 1 = 0$ or $9n^2 + 27n - 1 = 0$

$a = -9,\ b = -27,\ c = 1$ $\quad$ $a = 9,\ b = 27,$ and $c = -1$

2. Solve each of the following quadratic equations by completing the square.

(3)

a. $x^2 - 6x + 5 = 0$

$x^2 - 6x = -5$

$x^2 - 6x + 9 = -5 + 9$

$(x - 3)^2 = 4$

$\sqrt{(x-3)^2} = \pm\sqrt{4}$

$x - 3 = \pm 2$

$x - 3 = 2$ or $x - 3 = -2$

$x = 5$ $\quad$ $x = 1$

The possible roots are 5 and 1.

(3) b. $-2x^2+8x-3=0$

$$2x^2-8x+3=0$$
$$2x^2-8x=-3$$
$$2\left(x^2-4x\right)=-3$$
$$2\left(x^2-4x+4\right)=-3+8$$
$$2(x-2)^2=5$$
$$(x-2)^2=\frac{5}{2}$$
$$\sqrt{(x-2)^2}=\pm\sqrt{\frac{5}{2}}$$
$$x-2=\pm\sqrt{\frac{5}{2}}$$
$$x=2\pm\sqrt{\frac{5}{2}}$$
$$x=2+\sqrt{\frac{5}{2}} \quad \text{or} \quad x=2-\sqrt{\frac{5}{2}}$$

The possible roots are $2+\sqrt{\frac{5}{2}}$ and $2-\sqrt{\frac{5}{2}}$.

3. Solve each of the following quadratic equations by using the quadratic formula. Express your answer in simplest radical form if the roots are not rational numbers.

(3) a. $5x^2 - 6x - 2 = 0$

$a = 5,\ b = -6,$ and $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{6 \pm \sqrt{36 + 40}}{10}$$

$$x = \frac{6 \pm \sqrt{76}}{10}$$

$$x = \frac{6 \pm 2\sqrt{19}}{10}$$

$$x = \frac{3 \pm \sqrt{19}}{5}$$

$$x = \frac{3 + \sqrt{19}}{5} \quad \text{or} \quad x = \frac{3 - \sqrt{19}}{5}$$

The possible roots are $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$.

(3) b. $3x^2 = -2(x+2)$

$$3x^2 = -2x - 4$$

$3x^2 + 2x + 4 = 0$

$a = 3,\ b = 2,$ and $c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 - 48}}{6}$$

$$x = \frac{-2 \pm \sqrt{-44}}{6}$$

There are no real number roots for this equation. The roots are complex numbers.

4. For each equation below, find the solutions correct to four decimal places.

(3) a. $4a^2 + 2a - 3 = 0$

$a = 4,\ b = 2,$ and $c = -3$

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4(4)(-3)}}{2(4)}$$

$$a = \frac{-2 \pm \sqrt{4 + 48}}{8}$$

$$a = \frac{-2 \pm \sqrt{52}}{8}$$

$$a = \frac{-2 \pm 2\sqrt{13}}{8}$$

$$a = \frac{-1 \pm \sqrt{13}}{4}$$

$$a = \frac{-1 + \sqrt{13}}{4} \quad \text{or} \quad a = \frac{-1 - \sqrt{13}}{4}$$

$$a \doteq \frac{-1 + 3.6056}{4} \qquad a \doteq \frac{-1 - 3.6056}{4}$$

$$a \doteq \frac{2.6056}{4} \qquad a \doteq \frac{-4.6056}{4}$$

$$a \doteq 0.6514 \qquad a \doteq -1.1514$$

The possible roots are approximately 0.6514 and -1.1514.

③ b. $2(x^2+5)-13x=0$

$2x^2+10-13x=0$

$2x^2-13x+10=0$

$a=2,\ b=-13,\ c=10$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-13)\pm\sqrt{(-13)^2-4(2)(10)}}{2(2)}$$

$$x=\frac{13\pm\sqrt{169-80}}{4}$$

$$x=\frac{13\pm\sqrt{89}}{4}$$

$$x=\frac{13+\sqrt{89}}{4} \quad \text{or} \quad x=\frac{13-\sqrt{89}}{4}$$

$$x\doteq\frac{13+9.4340}{4} \qquad x\doteq\frac{13-9.4340}{4}$$

$$x\doteq\frac{22.434}{4} \qquad x\doteq\frac{3.566}{4}$$

$x\doteq 5.6085$ $\qquad$ $x\doteq 0.8915$

The possible roots are approximately 5.6085 and 0.8915.

5. Solve each of the following quadratic equations using any method.

(2) a. $2x^2 - 16x + 32 = 0$

$2(x^2 - 8x + 16) = 0$

$2(x-4)(x-4) = 0$

$2 \neq 0;\ x - 4 = 0$

$x = 4$

The solution is $x = 4$.

(3) b. $4x^2 + 32x - 60 = 0$

$4(x^2 + 8x - 15) = 0$

$a = 1,\ b = 8,$ and $c = -15$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-15)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{64 + 60}}{2}$$

$$x = \frac{-8 \pm \sqrt{124}}{2}$$

$$x = \frac{-8 \pm 2\sqrt{31}}{2}$$

$$x = -4 \pm \sqrt{31}$$

$x = -4 + \sqrt{31}$ or $x = -4 - \sqrt{31}$

The possible roots are $x = -4 + \sqrt{31}$ and $x = -4 - \sqrt{31}$.

(3) c. $3x^2+12x+5=0$

$a=3,\ b=12,$ and $c=5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-12\pm\sqrt{12^2-4(3)(5)}}{2(3)}$$

$$x=\frac{-12\pm\sqrt{144-60}}{6}$$

$$x=\frac{-12\pm\sqrt{84}}{6}$$

$$x=\frac{-12\pm2\sqrt{21}}{6}$$

$$x=\frac{-6\pm\sqrt{21}}{3}$$

$$x=\frac{-6+\sqrt{21}}{3} \quad \text{or} \quad x=\frac{-6-\sqrt{21}}{3}$$

The possible roots are $\frac{-6+\sqrt{21}}{3}$ and $\frac{-6-\sqrt{21}}{3}$.

(2) d. $3x^2+8x-3=0$

$$(3x-1)(x+3)=0$$

$$3x-1=0 \quad \text{or} \quad x+3=0$$

$$3x=1 \qquad\qquad x=-3$$

$$x=\frac{1}{3}$$

The possible roots are $\frac{1}{3}$ and -3.

Topic 3

_____ marks

Topic 4: The Discriminant

(4)

1. Find the discriminant for each of the following equations.

a. $2x^2 - 7x - 4 = 0$

$a = 2,\ b = -7,$ and $c = -4$

$D = b^2 - 4ac$

$D = (-7)^2 - 4(2)(-4)$

$D = 49 + 32$

$D = 81$

The discriminant is 81.

b. $5a^2 + 4a + 3 = 0$

$a = 5,\ b = 4,$ and $c = 3$

$D = b^2 - 4ac$

$D = 4^2 - 4(5)(3)$

$D = 16 - 60$

$D = -44$

The discriminant is – 44.

(6)

2. Find the discriminant for each quadratic equation and describe the nature of the roots for each equation.

a. $4x^2 - 20x + 25 = 0$

$a = 4,\ b = -20,$ and $c = 25$

$D = b^2 - 4ac$

$D = (-20)^2 - 4(4)(25)$

$D = 400 - 400$

$D = 0$

The roots are real and equal.

b. $9x^2 - 5x - 2 = 0$

$a = 9,\ b = -5,$ and $c = -2$

$D = b^2 - 4ac$

$D = (-5)^2 - 4(9)(-2)$

$D = 25 + 72$

$D = 97$

The roots are real, unequal, and irrational.

c. $4y^2 + 7y + 5 = 0$

$a = 4,\ b = 7,$ and $c = 5$

$D = b^2 - 4ac$

$D = 7^2 - 4(4)(5)$

$D = 49 - 80$

$D = -31$

The roots are not real.

Topic 4

_____ marks

Topic 5: Applications of Quadratic Equations.

(3) 1. The dimensions of a garden plot are given by the roots of the quadratic equation $a^2 - 18a + 72 = 0$. What are the dimensions of the garden plot in metres?

$$a^2 - 18a + 72 = 0$$
$$(a-12)(a-6) = 0$$
$$a - 12 = 0 \quad \text{or} \quad a - 6 = 0$$
$$a = 12 \qquad\qquad a = 6$$

The garden plot is 12 m by 6 m.

(3) 2. During the 1984 Summer Olympics in Los Angeles, Canada won a number of gold and silver medals. Solve the equation $3x^2 - 84x + 540 = 0$ to determine the actual number of silver and gold medals won. Once the solution for x is found, the larger value will be silver medals and the smaller value will be gold medals

$$3x^2 - 84x + 540 = 0$$
$$3\left(x^2 - 28x + 180\right) = 0$$
$$3(x-18)(x-10) = 0$$
$$3 \neq 0;\ x - 18 = 0 \quad \text{or} \quad x - 10 = 0$$
$$x = 18 \qquad\qquad x = 10$$

Canada won 18 silver medals and 10 gold medals at the 1984 Summer Olympics.

④ 3. Two square photographs are to be printed on the same page. A side of the larger photograph is 5 cm longer than a side of the smaller one. If both photos cover a total area of 157 cm^2, find the length of a side of each photo.

Let a side of the smaller photo be x cm.
Thus, a side of the larger photo is $(x+5)$ cm.

$$x^2+(x+5)^2=157$$
$$x^2+x^2+10+25=157$$
$$2x^2+10x+25=157$$
$$2x^2+10x-132=0$$
$$2\left(x^2+5x-66\right)=0$$
$$2(x+11)(x-6)=0$$
$$2\neq 0;\ x+11=0 \quad \text{or} \quad x-6=0$$
$$x=-11 \qquad x=6$$

The root $x=-11$ is rejected since a measure of length can not be negative. Therefore, a side of the smaller square is 6 cm and a side of the larger square is $x+5=6+5=11$ cm.

④ 4. Find the three consecutive odd integers whose sum of their squares is 875.

Let the first interger be x.
The next integer will be $(x+2)$ or $(x-2)$ and the third one will be $(x+4)$ or $(x-4)$.

$$x^2+(x+2)^2+(x+4)^2=875$$
$$x^2+x^2+4x+4+x^2+8x+16=875$$
$$3x^2+12x+20=875$$
$$3x^2+12x-855=0$$
$$3\left(x^2+4x-285\right)=0$$
$$3(x+19)(x-15)=0$$
$$3\neq 0;\ x+19=0 \quad \text{or} \quad x-15=0$$
$$x=-19 \qquad x=15$$

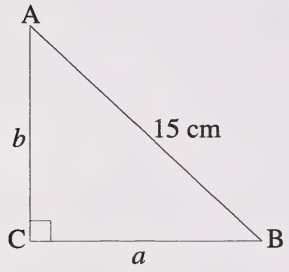
Two sets of integers are possible: -15, -17, and -19 or 15, 17, and 19.

Using $x^2+(x-2)^2+(x-4)^2=875$, you would get

$$x^2+x^2-4x+4+x^2-8x+16=875$$
$$3x^2-12x+20=875$$
$$3x^2-12x-855=0$$
$$3\left(x^2-4x-285\right)=0$$
$$3(x-19)(x+15)=0$$
$$3\neq 0;\ x-19=0 \quad \text{or} \quad x+15=0$$
$$x=19 \qquad x=-15$$

The three integers are 15, 17, and 19 or -15, -17, and -19.

(3) 5. The distance around a right triangle is 36 cm. If the side opposite the right angle is 15 cm, what is the length of the other two sides?



$$P = a + b + c$$
$$36 = a + b + 15$$
$$a + b = 36 - 15$$
$$a + b = 21$$
$$a = 21 - b$$

$$a^2 + b^2 = c^2$$
$$(21-b)^2 + b^2 = 15^2$$
$$441 - 42b + b^2 + b^2 = 225$$
$$2b^2 - 42b + 216 = 0$$
$$2\left(b^2 - 21b + 108\right) = 0$$
$$2(b-9)(b-12) = 0$$
$$2 \neq 0;\ b-9=0 \quad \text{or} \quad b-12=0$$
$$b = 9 \qquad\qquad b = 12$$

If $b = 9$, then $a = 21 - 9 = 12$.
If $b = 12$, then $a = 21 - 12 = 9$.
The lengths of the other two sides are 12 cm and 9 cm.

6. Solve each of the following rational equations. Check each solution.

(3) a. $\dfrac{3m}{m-3}+2=\dfrac{3m-1}{m+3} \quad m\neq 3,\ m\neq -3$

$\dfrac{3m}{m-3}+2=\dfrac{3m-1}{m+3} \quad \text{L.C.D}=(m-3)(m+3)$

$$(\cancel{m-3})(m+3)\left(\frac{3m}{\cancel{m-3}}\right)+(m-3)(m+3)(2)=(m-3)(\cancel{m+3})\left(\frac{3m-1}{\cancel{m+3}}\right)$$

$$3m(m+3)+2\left(m^2-9\right)=(m-3)(3m-1)$$

$$3m^2+9m+2m^2-18=3m^2-10m+3$$

$$2m^2+19m-21=0$$

$$(2m+21)(m-1)=0$$

$$2m+21=0 \quad \text{or} \quad m-1=0$$

$$2m=-21 \qquad m=1$$

$$m=\frac{-21}{2}$$

The posible roots are $\frac{-21}{2}$ and 1.

Check for $m = \frac{-21}{2}$.

LS	RS
$\frac{3m}{m-3}+2$	$\frac{3m-1}{m+3}$
$=\frac{3\left(\frac{-21}{2}\right)}{\frac{-21}{2}-3}+2$	$=3\frac{\left(\frac{-21}{2}\right)-1}{\frac{-21}{2}+3}$
$=\frac{\frac{-63}{2}}{\frac{-21}{2}-\frac{6}{2}}+2$	$=\frac{\frac{-63}{2}-1}{\frac{-21}{2}+\frac{6}{2}}$
$=\frac{\frac{-63}{2}}{\frac{-27}{2}}+2$	$=\frac{\frac{-63}{2}-\frac{2}{2}}{\frac{-15}{2}}$
$=\frac{-63}{2}\div\frac{-27}{2}+2$	$=\frac{\frac{-65}{2}}{\frac{-15}{2}}$
$=\frac{-63}{2}\times\frac{-2}{27}+2$	$=\frac{-65}{2}\div\frac{-15}{2}$
$=\frac{63}{27}+2$	$=\frac{-65}{2}\times\frac{-2}{15}$
$=2\frac{1}{3}+2$	$=\frac{65}{15}$
$=4\frac{1}{3}$	$=4\frac{1}{3}$

LS = RS

Check for $m = 1$.

LS	RS
$=\frac{3m}{m-3}+2$	$\frac{3m-1}{m+3}$
$=\frac{3(1)}{1-3}+2$	$=\frac{3(1)-1}{1+3}$
$=\frac{3}{-2}+2$	$=\frac{3-1}{4}$
$=\frac{-3}{2}+\frac{4}{2}$	$=\frac{2}{4}$
$=\frac{1}{2}$	$=\frac{1}{2}$

LS = RS

The roots are $\frac{-21}{2}$ and 1.

③ b. $\dfrac{2y}{y-2}=\dfrac{10}{y+2}+\dfrac{y^2+3y}{y^2-4}$ $\qquad y\neq\pm 2$

$$\frac{2y}{y-2}=\frac{10}{y+2}+\frac{y^2+3y}{y^2-4} \qquad \text{L.C.D.}=(y+2)(y-2)=y^2-4$$

$$(y+2)\cancel{(y-2)}\left(\frac{2y}{\cancel{y-2}}\right)=\cancel{(y+2)}(y-2)\left(\frac{10}{\cancel{y+2}}\right)+\cancel{(y^2-4)}\left(\frac{y^2+3y}{\cancel{y^2-4}}\right)$$

$$2y(y+2)=10(y-2)+(y^2+3y)$$

$$2y^2+4y=10y-20+y^2+3y$$

$$2y^2+4y=13y-20+y^2$$

$$y^2-9y+20=0$$

$$(y-4)(y-5)=0$$

$$y-4=0 \quad \text{or} \quad y-5=0$$

$$y=4 \qquad y=5$$

The possible roots are 4 and 5.

Check for $y=4$.

LS	RS
$\dfrac{2y}{y-2}$	$\dfrac{10}{y+2}+\dfrac{y^2+3y}{y^2-4}$
$=\dfrac{2(4)}{4-2}$	$=\dfrac{10}{4+2}+\dfrac{4^2+3(4)}{4^2-4}$
$=\dfrac{8}{2}$	$=\dfrac{10}{6}+\dfrac{16+12}{16-4}$
$=4$	$=\dfrac{10}{6}+\dfrac{28}{12}$
	$=\dfrac{20+28}{12}$
	$=\dfrac{48}{12}$
	$=4$
LS = RS	

Check for $y=5$.

LS	RS
$\dfrac{2y}{y-2}$	$\dfrac{10}{y+2}+\dfrac{y^2+3y}{y^2-4}$
$=\dfrac{2(5)}{5-2}$	$=\dfrac{10}{5+2}+\dfrac{5^2+3(5)}{5^2-4}$
$=\dfrac{10}{3}$	$=\dfrac{10}{7}+\dfrac{25+15}{25-4}$
	$=\dfrac{10}{7}+\dfrac{40}{21}$
	$=\dfrac{30+40}{21}$
	$=\dfrac{70}{21}$
	$=\dfrac{10}{3}$
LS = RS	

The roots are 4 and 5.

③ c. $\dfrac{5y}{y+2}-2=\dfrac{y}{y-2}-\dfrac{3y+1}{y^2-4}$ $\qquad y\neq\pm 2$

$\dfrac{5y}{y+2}-2=\dfrac{y}{y-2}-\dfrac{3y+1}{y^2-4}$ $\qquad$ L.C.D. $=(y+2)(y-2)=y^2-4$

$$\overset{1}{\cancel{(y+2)}}(y-2)\left(\frac{5y}{\cancel{y+2}_{1}}\right)-(y+2)(y-2)(2)=(y+2)\overset{1}{\cancel{(y-2)}}\left(\frac{y}{\cancel{y-2}_{1}}\right)-\overset{1}{\cancel{(y+2)}}\overset{1}{\cancel{(y-2)}}\left(\frac{3y+1}{\cancel{(y+2)}_{1}\cancel{(y-2)}_{1}}\right)$$

$$5y(y-2)-2\left(y^2-4\right)=y(y+2)-(3y+1)$$

$$5y^2-10y-2y^2+8=y^2+2y-3y-1$$

$$3y^2-10y+8=y^2-y-1$$

$$2y^2-9y+9=0$$

$$(2y-3)(y-3)=0$$

$$2y-3=0 \quad \text{or} \quad y-3=0$$

$$2y=3 \qquad\qquad y=3$$

$$y=\frac{3}{2}$$

The possible roots are $\frac{3}{2}$ and 3.

Check for $y=\frac{3}{2}$.

LS	RS
$\frac{5y}{y+2}-2$	$\frac{y}{y-2}-\frac{3y+1}{y^2-4}$
$=\frac{5\left(\frac{3}{2}\right)}{\frac{3}{2}+2}-2$	$=\frac{\frac{3}{2}}{\frac{3}{2}-2}-\frac{3\left(\frac{3}{2}\right)+1}{\left(\frac{3}{2}\right)^2-4}$
$=\frac{\frac{15}{2}}{\frac{3}{2}+\frac{4}{2}}-2$	$=\frac{\frac{3}{2}}{\frac{3}{2}-\frac{4}{2}}-\frac{\frac{9}{2}+1}{\frac{9}{4}-4}$
$=\frac{\frac{15}{2}}{\frac{7}{2}}-2$	$=\frac{\frac{3}{2}}{\frac{-1}{2}}-\frac{\frac{9}{2}+\frac{2}{2}}{\frac{9}{4}-\frac{16}{4}}$
$=\frac{15}{2}\div\frac{7}{2}-2$	$=\frac{3}{2}\div\frac{-1}{2}-\frac{\frac{11}{2}}{\frac{-7}{4}}$
$=\frac{15}{2}\times\frac{2}{7}-2$	$=\frac{3}{2}\times\frac{-2}{1}-\left(\frac{11}{2}\div\frac{-7}{4}\right)$
$=\frac{15}{7}-2$	$=-3-\left(\frac{11}{2}\times\frac{-4}{7}\right)$
$=\frac{15}{7}-\frac{14}{7}$	$=-3-\left(\frac{-22}{7}\right)$
$=\frac{1}{7}$	$=\frac{-21}{7}+\frac{22}{7}$
	$=\frac{1}{7}$
LS = RS	

Check for $y=3$.

LS	RS
$\frac{5y}{y+2}-2$	$\frac{y}{y-2}-\frac{3y+1}{y^2-4}$
$=\frac{5(3)}{3+2}-2$	$=\frac{3}{3-2}-\frac{3(3)+1}{3^2-4}$
$=\frac{15}{5}-2$	$=\frac{3}{1}-\frac{9+1}{9-4}$
$=3-2$	$=3-\frac{10}{5}$
$=1$	$=3-2$
	$=1$
LS = RS	

The roots are $\frac{3}{2}$ and 3.

7. Solve each of the following radical equations. Check each root and reject extraneous roots.

(3) a. $\sqrt{2t^2-2}-t=1$

$$\sqrt{2t^2-2}=t+1$$
$$\left(\sqrt{2t^2-2}\right)^2=(t+1)^2$$
$$2t^2-2=t^2+2t+1$$
$$t^2-2t-3=0$$
$$(t-3)(t+1)=0$$
$$t-3=0 \quad \text{or} \quad t+1=0$$
$$t=3 \qquad\qquad t=-1$$

The possible roots are 3 and -1.

Check for $t=3$.

LS	RS
$\sqrt{2t^2-2}-t$	1
$=\sqrt{2(3)^2-2}-3$	
$=\sqrt{2(9)-2}-3$	
$=\sqrt{18-2}-3$	
$=4-3$	
$=1$	
LS = RS	

Check for $t=-1$.

LS	RS
$\sqrt{2t^2-2}-t$	1
$=\sqrt{2(-1)^2-2}-(-1)$	
$=\sqrt{2(1)-2}+1$	
$=\sqrt{2-2}+1$	
$=\sqrt{0}+1$	
$=0+1$	
$=1$	
LS = RS	

The solutions are 3 and -1.

(3) b. $\sqrt{x-1}+\sqrt{3x+1}=6$

$$\sqrt{x-1}=6-\sqrt{3x+1}$$
$$\left(\sqrt{x-1}\right)^2=\left(6-\sqrt{3x+1}\right)^2$$
$$x-1=36-12\sqrt{3x+1}+3x+1$$
$$x-1=37+3x-12\sqrt{3x+1}$$
$$-2x-38=-12\sqrt{3x+1}$$
$$(-2x-38)^2=\left(-12\sqrt{3x+1}\right)^2$$
$$4x^2+152x+1444=144(3x+1)$$
$$4x^2+152x+1444=432x+144$$
$$4x^2-280x+1300=0$$
$$4\left(x^2-70x+325\right)=0$$
$$4(x-65)(x-5)=0$$
$$4\neq 0;\ x-65=0 \quad \text{or} \quad x-5=0$$
$$x=65 \qquad\qquad x=5$$

The possible roots are 65 and 5.

Check for $x=65$.

LS	RS
$\sqrt{x-1}+\sqrt{3x+1}$	6
$=\sqrt{65-1}+\sqrt{3(65)+1}$	
$=\sqrt{64}+\sqrt{195+1}$	
$=8+\sqrt{196}$	
$=8+14$	
$=22$	
LS ≠ RS	

Check for $x=5$.

LS	RS
$\sqrt{x-1}+\sqrt{3x+1}$	6
$=\sqrt{5-1}+\sqrt{3(5)+1}$	
$=\sqrt{4}+\sqrt{15+1}$	
$=2+\sqrt{16}$	
$=2+4$	
$=6$	
LS = RS	

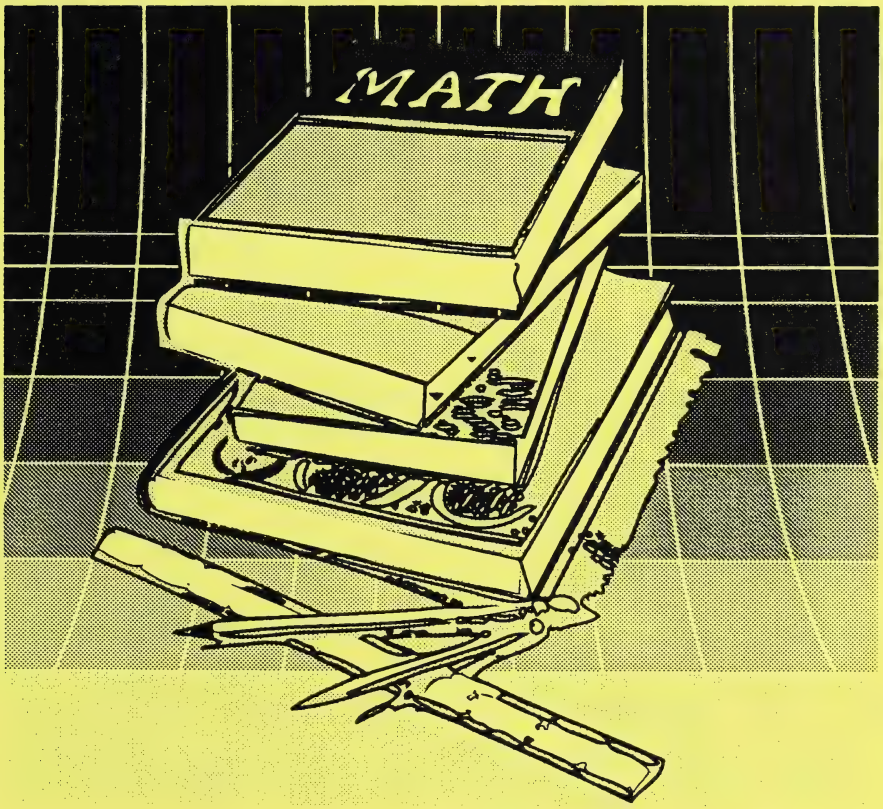
The root is $x=5$. The root $x=65$ is an extraneous root.

Topic 5

_____ marks

UNIT 7

MATHEMATICS 20



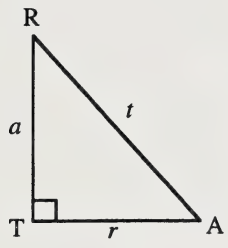
LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

Topic 1: Introduction to Trigonometry

(2) 1. Use R, A, and T to label the given right triangle. The right angle is T. Using the two acute angles, give the primary trigonometric ratios for each angle.



For $\angle R$,

$\sin R = \frac{r}{t}$

$\cos R = \frac{a}{t}$

$\tan R = \frac{r}{a}$

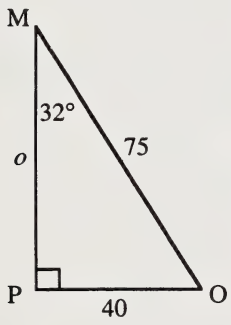
For $\angle A$,

$\sin A = \frac{a}{t}$

$\cos A = \frac{r}{t}$

$\tan A = \frac{a}{r}$

(2) 2. Use the Theorem of Pythagoras to solve right triangle MOP. Round the answers to the nearest whole number. Provide a summary.



$\angle O = 180° - (90° + 32°)$

$\angle O = 180° - 122°$

$\angle O = 58°$

$o^2 = 75^2 - 40^2$

$o^2 = 5625 - 1600$

$o^2 = 4025$

$\sqrt{o^2} = \sqrt{4025}$

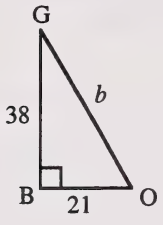
$o = 63.443$

$o = 63$

Summary

$\angle M = 32°$	$m = 40$
$\angle O = 58°$	$o = 63$
$\angle P = 90°$	$p = 75$

(2) 3. Use the Theorem of Pythagoras and trigonometric ratios to solve ΔBOG. Round the answers to the nearest whole number. Provide a summary.



$$\tan O = \frac{38}{21}$$
$$\tan O = 1.8095$$
$$\angle O = 61°$$

$$\angle G = 180° - (90° + 61°)$$
$$\angle G = 180° - 151°$$
$$\angle G = 29°$$

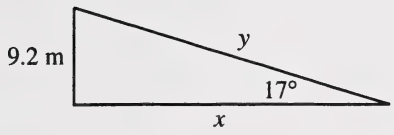
$$b^2 = 38^2 + 21^2$$
$$b^2 = 1444 + 441$$
$$b^2 = 1885$$
$$\sqrt{b^2} = \sqrt{1885}$$
$$b = 43.417$$
$$b = 43$$

Summary

$\angle G = 29°$	$m = 21$
$\angle O = 61°$	$o = 38$
$\angle B = 90°$	$p = 43$

4. An underground parking lot is being built at a high-rise apartment block. This lot is 9.2 m below ground level.

(2) a. If the exit ramp is to rise at an angle of 17°, what will the length of the ramp be? (Answer to the nearest tenth of a metre.)



Let x be the length of the ramp.

$$\sin 17° = \frac{9.2}{x}$$

$$0.2924 = \frac{9.2}{x}$$

$$x = 9.2 \div 0.2924$$

$$x = 31.464$$

$$x = 31.5$$

The length of the ramp is about 31.5 m.

(2) b. What is the horizontal distance needed for the ramp? (Answer to the nearest tenth of a metre.)

Let y be the horizontal distance that is needed for the ramp.

$$\tan 17° = \frac{9.2}{y}$$

$$0.3057 = \frac{9.2}{y}$$

$$y = 9.2 \div 0.3057$$

$$y = 30.095$$

$$y = 30.1$$

The horizontal distance needed for the ramp is approximately 30.1 m.

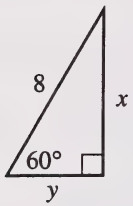
Topic 1

_____ marks

Topic 2: Special Triangles

(12) 1. Where necessary, use radical expressions to find the exact measures of the unknown sides in each triangle.

a.



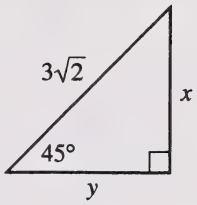
y is one-half of the hypotenuse.

$y = \frac{1}{2} \times 8 = 4$

x is y multiplied by $\sqrt{3}$.

$x = 4\sqrt{3}$

b.

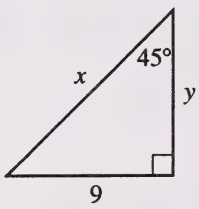


$x = y$ since this is an isosceles triangle.

$x = 3$

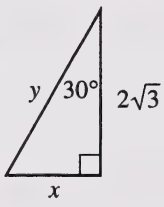
$y = 3$

c.



This triangle is an isosceles triangle so $y = 9$ and $x = 9\sqrt{2}$.

d.



$x = 2$ and $y = 2 \times 2 = 4$

2. Find the value for each of the following. Each answer should be in simplified radical form or any other simplified form.

(1) a. $(\sin 45°)(\cos 45°)+(\cos 30°)(\tan 60°)$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}\right)$$
$$=\frac{\sqrt{4}}{4}+\frac{\sqrt{9}}{2}$$
$$=\frac{2}{4}+\frac{3}{2}$$
$$=\frac{1}{2}+\frac{3}{2}$$
$$=\frac{4}{2}$$
$$=2$$

(1) b. $(\sin 30°+\tan 45°)+(\cos 60°+\sin 30°)$

$$\left(\frac{1}{2}+1\right)+\left(\frac{1}{2}+\frac{1}{2}\right)$$
$$=\frac{3}{2}+\frac{2}{2}$$
$$=\frac{5}{2}$$
$$=2\frac{1}{2}$$

(1) c. $(\cos 30°)(\tan 30°)+(\sin 60°)(\tan 60°)$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{3}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}\right)$$
$$=\frac{\sqrt{9}}{6}+\frac{\sqrt{9}}{2}$$
$$=\frac{3}{6}+\frac{3}{2}$$
$$=\frac{1}{2}+\frac{3}{2}$$
$$=\frac{4}{2}$$
$$=2$$

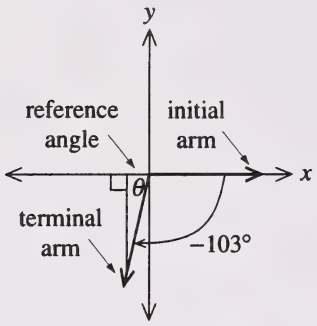
Topic 2

_____ marks

Topic 3: Angles on a Coordinate System

1. Draw each angle in standard position. Show the initial arm and the terminal arm for each angle. Determine the reference angle and give its measure.

(2) a. $-103°$

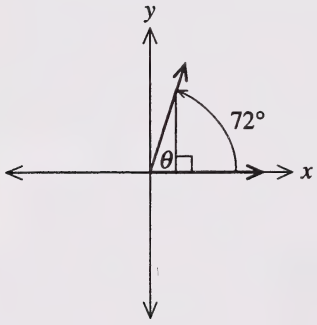


θ is the reference angle.

$\theta = 180° - 103°$

$\theta = 77°$

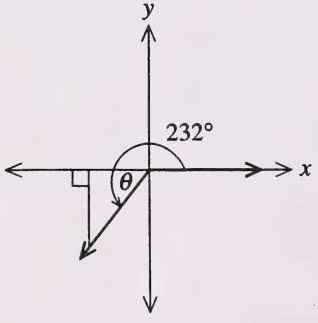
(2) b. $72°$



θ is the reference angle.

$\theta = 72°$, since any angle less than $90°$ is its own reference angle.

(2) c. $232°$

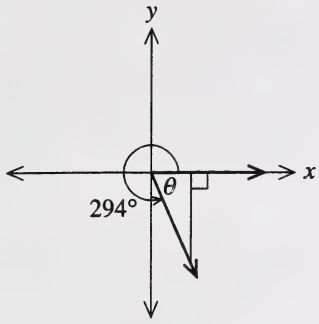


θ is the reference angle.

$\theta = 232° - 180°$

$\theta = 52°$

(2) d. 294°



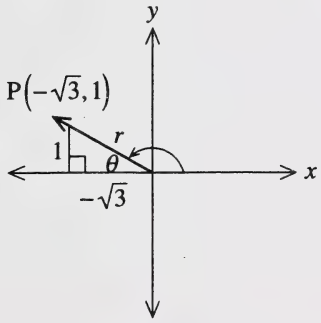
θ is the reference angle.

$\theta = 360° - 294°$

$\theta = 66°$

2. What is the measure of the positive angle whose terminal arm passes through the following points?

(2) a. $P(-\sqrt{3}, 1)$



$r^2 = 1^2 + (-\sqrt{3})^2$

$r^2 = 1 + 3$

$r^2 = 4$

$r = 2$

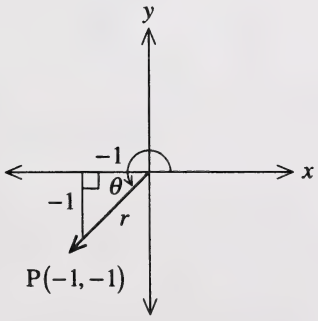
$\sin \theta = \frac{1}{2}$

$\sin \theta = 0.5000$

$\theta = 30°$

The angle measure is $180° - 30° = 150°$.

(2) b. $P(-1, -1)$



$\tan \theta = \frac{-1}{-1}$

$\tan \theta = 1$

$\theta = 45°$

The measure of the angle is $180° + 45° = 225°$.

(1) 3. a. What positive angle terminating in the third quadrant has a reference angle of 55°?

Let the positive angle be θ.

$$\theta = 180° + 55° = 235°$$

(1) b. What negative angle terminating in the second quadrant has a reference angle of 25°?

Let the negative angle be θ.

$$\theta = -(180° + 25°) = -205°$$

(1) c. Name two angles terminating in the fourth quadrant having a reference angle of 42°.

The negative angle is $-42°$.
The positive angle is $360° - 42° = 318°$.

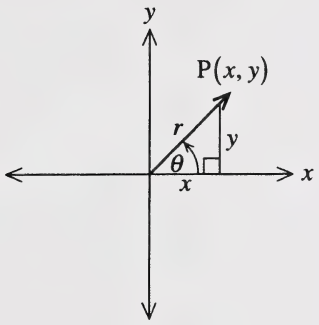
Topic 3

_____ marks

Topic 4: Trigonometric Ratios

(6) 1. Draw an angle in standard position in each quadrant. Determine the reference angle for each, and construct the appropriate triangle containing this reference angle. Label the sides of the triangles using x, y, and r. Using these sides, determine the trigonometric ratios for each angle in standard position.

Quadrant 1

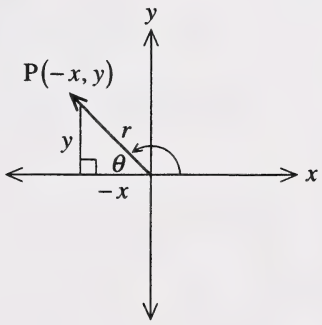


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

Quadrant 2

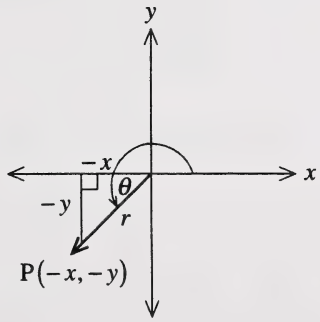


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{y}{-x}$ or $\frac{-y}{x}$

Quadrant 3

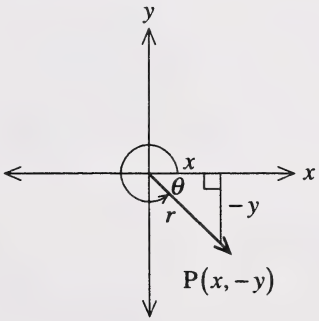


$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{-y}{-x} = \frac{y}{x}$

Quadrant 4



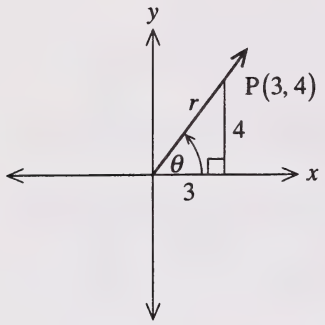
$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{-y}{x}$

2. What are the trigonometric ratios for angles whose terminal arms pass through the following points? Provide these ratios in simplest fraction form and in decimal number form rounded to four decimal places.

(3) a. $P(3,4)$



$r^2 = 4^2 + 3^2$

$r^2 = 16 + 9$

$r^2 = 25$

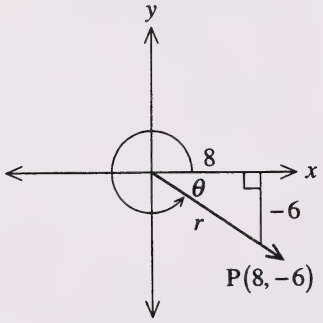
$r = 5$

$\sin\theta = \frac{4}{5} = 0.8000$

$\cos\theta = \frac{3}{5} = 0.6000$

$\tan\theta = \frac{4}{3} = 1.3333$

(3) b. $P(8,-6)$



$r^2 = (-6)^2 + 8^2$

$r^2 = 36 + 64$

$r^2 = 100$

$r = 10$

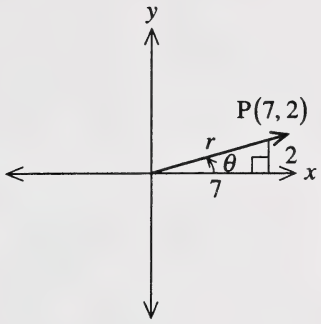
$\sin\theta = \frac{-6}{10} = -0.6000$

$\cos\theta = \frac{8}{10} = 0.8000$

$\tan\theta = \frac{-6}{8} = -0.7500$

3. Find the measure of the positive angle in standard position whose terminal arm passes through each of the following points. Express each angle measure to the nearest degree.

(2) a. $P(7, 2)$

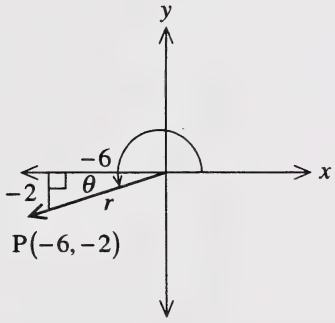


Use any of the three trigonometric ratios since they are all positive in Quadrant 1.

$$\tan\theta = \frac{2}{7}$$
$$\tan\theta = 0.2857$$
$$\theta = 16°$$

The angle measure is 16°.

(2) b. $P(-6, -2)$



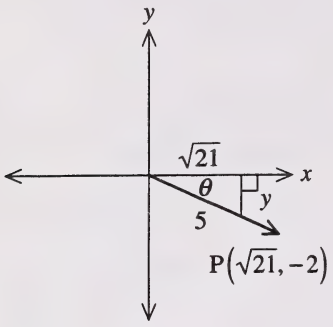
The tangent ratio is positive in Quadrant 3.

$$\tan\theta = \frac{-2}{-6}$$
$$\tan\theta = \frac{1}{3}$$
$$\tan\theta = 0.3333$$
$$\theta = 18°$$

The measure of the angle in standard position is $180° + 18° = 198°$.

4. For each trigonometric ratio, sketch a diagram of the angle in standard position, and find the value of the other trigonometric ratios.

(2) a. If $\cos\theta = \frac{\sqrt{21}}{5}$, θ terminates in Quadrant 4.



$y^2 = 5^2 - \left(\sqrt{21}\right)^2$

$y^2 = 25 - 21$

$y^2 = 4$

$y = \pm 2$

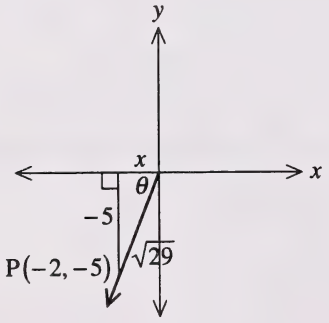
$y = -2$ (in Quadrant 4)

$\sin\theta = \frac{-2}{5} = -0.4000$

$\tan\theta = \frac{-2}{\sqrt{21}} = \frac{-2}{4.5826} = -0.4364$

$\tan\theta$ can also be $\frac{-2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$.

(2) b. If $\sin\theta = \frac{-5}{\sqrt{29}}$, θ terminates in Quadrant 3.



$x^2 = \left(\sqrt{29}\right)^2 - (-5)^2$

$x^2 = 29 - 25$

$x^2 = 4$

$x = \pm 2$

$x = -2$ (in Quadrant 3)

$\cos\theta = \frac{-2}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}$ or -0.3714

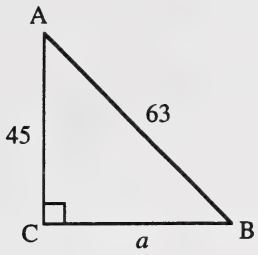
$\tan\theta = \frac{-5}{-2} = 2\frac{1}{2}$ or 2.5000

Topic 4

_____ marks

Topic 5: Solving Right Triangles

③ 1. Solve ΔABC. Round each measure to the nearest whole number. Provide a summary.



$$a^2 = 63^2 - 45^2$$
$$a^2 = 3969 - 2025$$
$$a^2 = 1944$$
$$a = 44$$

$$\sin \angle B = \frac{45}{63}$$
$$\sin \angle B = 0.7143$$
$$\angle B = 46°$$

$$\cos \angle A = \frac{45}{63}$$
$$\cos \angle A = 0.7143$$
$$\angle A = 44°$$

or

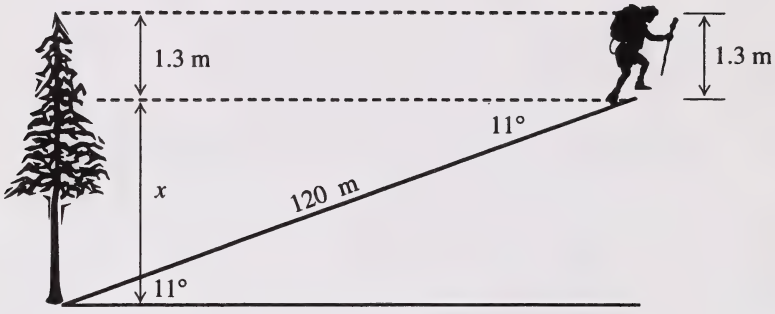
$$\angle A = 180° - (90° + 46°)$$
$$\angle A = 180° - 136°$$
$$\angle A = 44°$$

Summary

$\angle A = 44°$ $\quad a = 44$
$\angle B = 46°$ $\quad b = 45$
$\angle C = 90°$ $\quad c = 63$

④ 2. Jackie is hiking up an 11° slope. She counts her steps from the base of a tree. After going 120 m up the slope, she notices that her eyes are on the same level as the top of the tree.

If her eyes are 1.3 m above the ground, how tall is the tree? (Answer correct to one decimal place.)



$$\sin 11° = \frac{x}{120}$$

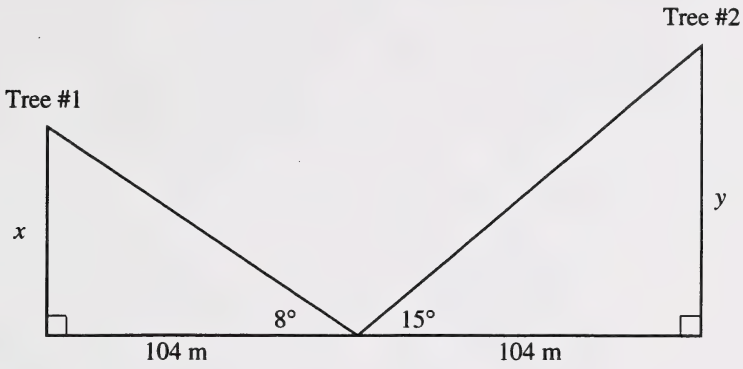
$$0.1908 = \frac{x}{120}$$

$$x = 120 \times 0.1908$$

$$x = 22.9$$

The height of the tree is $22.9 + 1.3 = 24.2$ m.

④ 3. Two trees are 208 m apart. From a point halfway between the two trees, the angles of elevation to the tops of the trees are 8° and 15°. How much taller is one tree than the other? (Answer correct to two decimal places.)



Tree #1

$$\tan 8° = \frac{x}{104}$$
$$x = 104 \times 0.140\,54$$
$$x = 14.616$$

Tree #1 is 14.62 m tall.

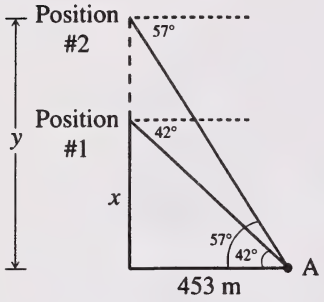
Tree #2

$$\tan 15° = \frac{y}{104}$$
$$y = 104 \times 0.2679$$
$$y = 27.86$$

Tree #2 is 27.86 m tall

Tree #2 is taller than tree #1 by $27.86 - 14.62 = 13.24$ m.

④ 4. The pilot of a helicopter which is hovering over a building picks point A on the ground at an angle of depression of 42°. Suddenly, the helicopter rises vertically, and the pilot sights point A now at an angle of depression of 57°. If point A is 453 m from the building, how many metres has the helicopter risen from its original height? (Write the answer correct to one decimal place.)



Step 1: Find the height of position #1 or distance x.

$$\tan 42° = \frac{x}{453}$$

$$0.9004 = \frac{x}{453}$$

$$x = 453 \times 0.9004$$

$$x = 407.9$$

The height of position #1 is 407.9 m.

Step 2: Find the height of position #2 or distance y.

$$\tan 57° = \frac{y}{453}$$

$$1.5399 = \frac{y}{453}$$

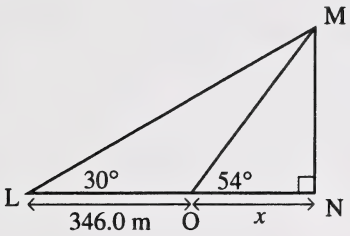
$$y = 453 \times 1.5399$$

$$y = 697.6$$

The height of position #2 is 697.6 m.

The helicopter rose $697.6 - 407.9 = 289.7$ m from its original height.

(5) 5. Use the given information to find the measure MN to the nearest tenth of a metre.



Step 1: In ΔLMN,

$$\tan 30° = \frac{MN}{346.0 + x}$$

$$0.5774 = \frac{MN}{346.0 + x}$$

$$MN = 0.5774(346.0 + x)$$

In ΔOMN,

$$\tan 54° = \frac{MN}{x}$$

$$1.3764 = \frac{MN}{x}$$

$$MN = 1.3764x$$

Since MN = MN,

$$1.3764x = 0.5774(346.0 + x)$$

$$1.3764x = 199.7804 + 0.5774x$$

$$0.7990x = 199.7804$$

$$x = 199.7804 \div 0.7990$$

$$x = 250.0$$

The entire length of LN is 346.0 + 250.0 = 596.0 m.

Step 2: To find MN, use ΔLMN or ΔOMN.

In ΔLMN,

$$\tan 30° = \frac{MN}{596.0}$$

$$0.5774 = \frac{MN}{596.0}$$

$$MN = 596.0 \times 0.5774$$

$$MN = 344.1$$

MN is about 344.1 m in length.

In ΔOMN,

$$\tan 54° = \frac{MN}{250.0}$$

$$1.3764 = \frac{MN}{250.0}$$

$$MN = 250 \times 1.3764$$

$$MN = 344.1$$

MN is about 344.1 m in length.

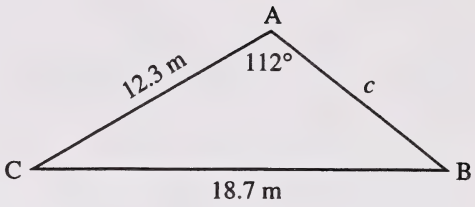
Topic 5

_____ marks

Topic 6: Sine and Cosine Laws

1. In ΔABC, $\angle A = 112°$, $b = 12.3$ m, and $a = 18.7$ m.

(2) a. Find $\angle B$ to the nearest degree.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{18.7}{\sin 112°} = \frac{12.3}{\sin B}$$
$$\frac{18.7}{0.9272} = \frac{12.3}{\sin B}$$
$$18.7 \sin B = 12.3 \times 0.9272$$
$$18.7 \sin B = 11.404\,56$$
$$\sin B = 11.404\,56 \div 18.7$$
$$\sin B = 0.6099$$
$$\angle B = 38°$$

To the nearest degree, $\angle B$ measures 38°.

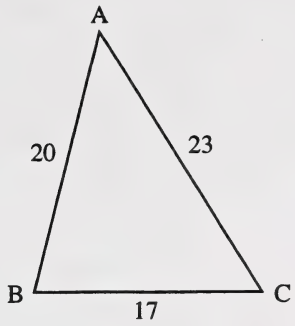
(2) b. Find c to one decimal place.

$$\angle C = 180° - (112° + 38°)$$
$$\angle C = 180° - 150°$$
$$\angle C = 30°$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{18.7}{\sin 112°} = \frac{c}{\sin 30°}$$
$$\frac{18.7}{0.9272} = \frac{c}{0.5}$$
$$0.9272c = 18.7 \times 0.5$$
$$0.9272c = 9.35$$
$$c = 9.35 \div 0.9272$$
$$c = 10.084$$
$$c = 10.1$$

To one decimal place, the measure of AB is 10.1 m.

(5) 2. In ΔABC, find the measures of the three angles if $a = 17$ cm, $b = 23$ cm, and $c = 20$ cm. Write the answers correct to the nearest degree.



To find $\angle A$, use

$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$17^2 = 23^2 + 20^2 - 2(23)(20)\cos A$$
$$289 = 529 + 400 - 920\cos A$$
$$289 = 929 - 920\cos A$$
$$920\cos A = 929 - 289$$
$$920\cos A = 640$$
$$\cos A = 640 \div 920$$
$$\cos A = 0.6957$$
$$\angle A = 46°$$

To find $\angle B$, use

$$b^2 = a^2 + c^2 - 2ac\cos B.$$
$$23^2 = 17^2 + 20^2 - 2(17)(20)\cos B$$
$$529 = 289 + 400 - 2(340)\cos B$$
$$529 = 689 - 680\cos B$$
$$680\cos B = 689 - 529$$
$$680\cos B = 160$$
$$\cos B = 160 \div 680$$
$$\cos B = 0.2353$$
$$\angle B = 76°$$

To find $\angle C$, use

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$20^2 = 17^2 + 23^2 - 2(17)(23)\cos C$$
$$400 = 289 + 529 - 2(391)\cos C$$
$$400 = 818 - 782\cos C$$
$$782\cos C = 818 - 400$$
$$782\cos C = 418$$
$$\cos C = 418 \div 782$$
$$\cos C = 0.5345$$
$$\angle C = 58°$$

or

$$\angle A + \angle B + \angle C = 180°$$
$$46° + 76° + \angle C = 180°$$
$$122° + \angle C = 180°$$
$$\angle C = 180° - 122°$$
$$\angle C = 58°$$

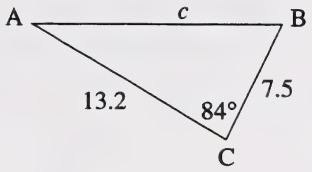
Summary

$\angle A = 46°$ $\quad a = 17$

$\angle B = 76°$ $\quad b = 23$

$\angle C = 58°$ $\quad c = 20$

③ 3. Solve ΔABC. Provide a summary.



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = (7.5)^2 + (13.2)^2 - 2(7.5)(13.2)\cos 84°$$
$$c^2 = 56.25 + 174.24 - 2(99)(0.1045)$$
$$c^2 = 230.49 - 198(0.1045)$$
$$c^2 = 230.49 - 20.691$$
$$c^2 = 209.799$$
$$\sqrt{c^2} = \sqrt{209.799}$$
$$c = 14.5$$

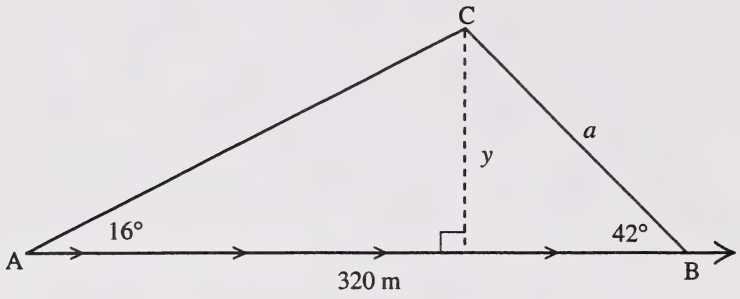
$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{14.5}{\sin 84°} = \frac{7.5}{\sin A}$$
$$\frac{14.5}{0.9945} = \frac{7.5}{\sin A}$$
$$14.5\sin A = 7.5 \times 0.9945$$
$$14.5\sin A = 7.458\,75$$
$$\sin A = 7.458\,75 \div 14.5$$
$$\sin A = 0.5144$$
$$\angle A = 31°$$

$$\angle B = 180° - (84° + 31°)$$
$$\angle B = 180° - 115°$$
$$\angle B = 65°$$

Summary

$\angle A = 31°$	$a = 7.5$
$\angle B = 65°$	$b = 13.2$
$\angle C = 84°$	$c = 14.5$

④ 4. While cycling along a straight road, Mark sees a car parked off the road at point C. The angle of the car's position to the road is 16°. After he goes for another 320 m to point B, the car is still at point C, and the angle is now 42°. When Mark is at point B, how far to the nearest metre is he from the car?



$\angle C = 180° - (16° + 42°)$

$\angle C = 180° - 58°$

$\angle C = 122°$

Use the Sine Law to solve for a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 16°} = \frac{320}{\sin 122°}$$

$$\frac{a}{0.2756} = \frac{320}{0.8480}$$

$$0.8480a = 320 \times 0.2756$$

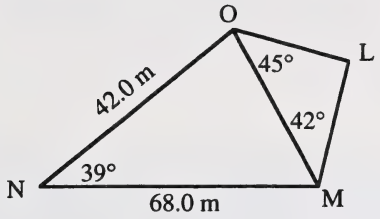
$$0.8480a = 88.192$$

$$a = \frac{88.192}{0.8480}$$

$$a = 104$$

When Mark is at point B, he is about 104 m from the car.

④ 5. How far is it from L to M? Express your answer to the nearest tenth of a metre.



Find the distance from O to M first.

$$n^2 = m^2 + o^2 - 2mo\cos N$$
$$n^2 = 42^2 + 68^2 - 2(42)(68)\cos 39°$$
$$n^2 = 1764 + 4624 - 2(2856)(0.7771)$$
$$n^2 = 6388 - 5712(0.7771)$$
$$n^2 = 6388 - 4438.7952$$
$$n^2 = 1949.2048$$
$$\sqrt{n^2} = \sqrt{1949.2048}$$
$$n = 44.1$$ (This is also side l for ΔLMO.)

The distance from O to M is about 44.1 m.

Find the measure of $\angle L$.

$$\angle L = 180° - (45° + 42°)$$
$$\angle L = 180° - 87°$$
$$\angle L = 93°$$

Last of all, find the measure of LM.

$$\frac{OM}{\sin L} = \frac{LM}{\sin O}$$

$$\frac{44.1}{\sin 93°} = \frac{LM}{\sin 45°}$$

$$\frac{44.1}{0.9986} = \frac{LM}{0.7071}$$

$$LM \times 0.9986 = 44.1 \times 0.7071$$

$$LM \times 0.9\,986 = 31.183\,409$$

$$LM = 31.183\,409 \div 0.9986$$

$$LM = 31.227$$

$$LM = 31.2$$

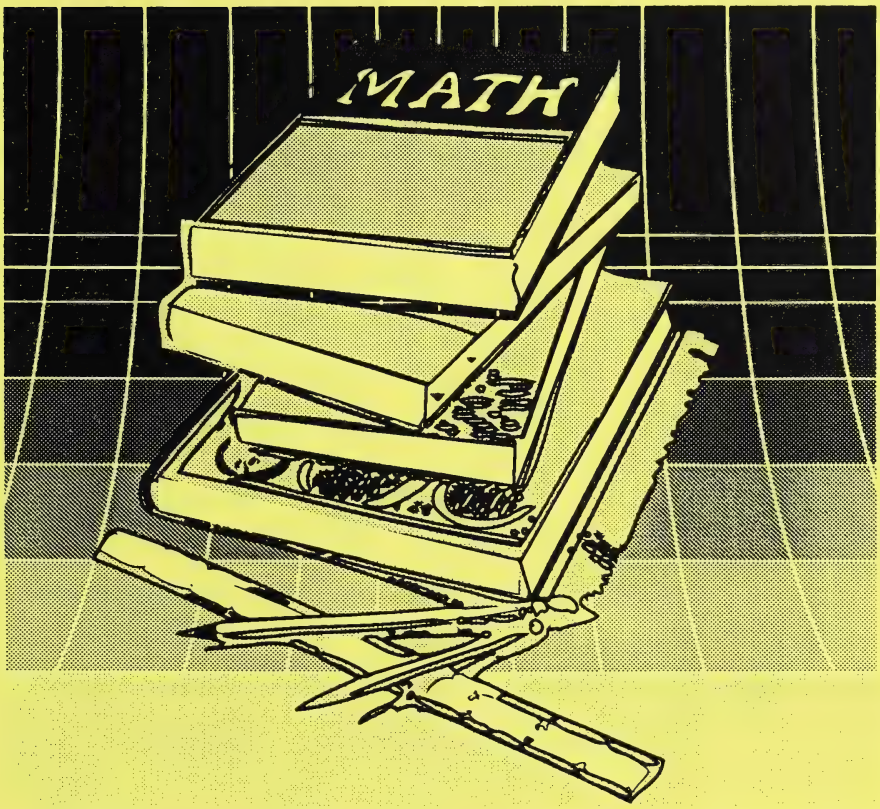
The distance from L to M is approximately 31.2 m.

Topic 6

_____ marks

UNIT 8

MATHEMATICS 20



LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

Topic 1: Probability

(5) 1. Conduct an experiment that involves choosing 3 balls at random from a box. (Make sure you record the colour and the order in which each ball was chosen.) The box contains 2 red balls, 1 green ball, and 2 blue balls. Since the order is important, at least 60 combinations will be required. From the recorded results of your experiment, determine the probability of choosing the combination blue–green–blue.

Answers will vary. A sample experiment is given.

Combination	Tally	Combination	Tally	Combination	Tally
blue–red–blue	III	green–red–red	III	red–green–blue	II
green–blue–red	IIII	red–blue–red	IIIIIII	red–blue–blue	II
red–red–green	II	red–green–red	IIIIII	blue–red–red	IIII
blue–green–red	IIIIIII	blue–blue–red	III	blue–green–blue	II
green–red–blue	III	red–blue–green	II	red–red–blue	I
blue–red–green	IIIIIII	blue–blue–green	II	green–blue–blue	

From the results of the experiment, $P(\text{blue} - \text{green} - \text{blue}) = \frac{2}{60} = \frac{1}{30}$.

(5) 2. Conduct an experiment that involves tossing a coin into the air 4 times and recording, in order, how it lands. Repeat the experiment at least 20 times. From the results of your experiment, determine the probability of obtaining two heads and two tails.

Answers will vary. A sample experiment is given.

Combination	4H	3H, 1T	2H, 2T	1H, 3T	4T
Tally	I	IIIIII	IIIII	IIIIIII	I

From the results of the experiment, $P(2\text{H},\ 2\text{T}) = \frac{5}{20} = \frac{1}{4}$.

(5)

3. Conduct an experiment that involves using only the kings and the queens from a deck of playing cards. From these 8 cards, deal 2 cards to one player and 3 cards to another. Repeat the experiment at least 30 times. From the results of your experiment, determine the probability of dealing the king of hearts and a black queen to the player with 2 cards.

Answers will vary. A sample experiment is given.

Trial	Combination 1st	2nd	Trial	Combination 1st	2nd	Trial	Combination 1st	2nd
1	Q♥	Q♣ K♦ K♣ Q♦	11	K♣	Q♣ K♦ Q♥ K♠	21	K♥	Q♥ K♣ Q♣ Q♠
2	K♣	Q♣ K♠ K♦ Q♥	12	Q♠	K♥ Q♣ Q♦ Q♥	22	K♠	K♦ Q♠ Q♣ K♥
3	K♣	Q♦ K♥ Q♣ K♦	13	K♦	K♥ Q♠ Q♣ K♣	23	K♥	K♣ Q♥ K♦ Q♦
4	Q♣	K♣ K♥ K♦ K♠	14	Q♠	K♦ K♥ Q♣ Q♥	24	K♦	Q♦ Q♥ Q♠ K♠
5	Q♦	K♦ Q♣ K♥ K♠	15	K♠	K♣ K♥ Q♦ Q♥	25	K♠	Q♦ Q♠ Q♣ K♦
6	K♣	Q♣ K♠ K♦ K♥	16	Q♠	K♥ K♣ Q♥ K♦	26	K♦	Q♣ Q♦ K♠ Q♥
7	KQ♥	K♥ K♣ Q♠	17	K♥	K♦ K♣ Q♠ Q♥	27	Q♣	Q♦ K♦ Q♥ K♣
8	Q♦	K♦ K♠ K♣ K♥	18	K♥	K♠ Q♥ Q♣ K♦	28	Q♦	Q♥ K♦ K♥ K♠
9	Q♥	Q♦ K♣ K♠ K♥	19	Q♣	K♠ Q♥ Q♠ K♦	29	Q♣	Q♦ K♥ Q♥ Q♠
10	K♣	K♥ Q♦ K♦ Q♥	20	K♠	K♥ Q♥ K♣ Q♣	30	K♠	Q♠ K♣ Q♥ Q♣

From the experiment, $P(K\heartsuit, Q_{black}) = \frac{2}{30} = \frac{1}{15}$.

(5) 4.

Number	Frequency Tally
2	||
3	||||
4	||||||
5	|||||
6	|||||||
7	|||||||||||||
8	|||||||||
9	|||||
10	|||||
11	|||
12	|

Two dice are rolled as part of a probability experiment and the results are recorded above. From the frequency table, determine the following. Express these as reduced fractions.

a. the probability of rolling a number greater than 4

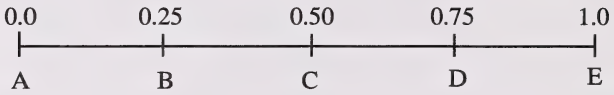
Probability of rolling a number > 4 = $\frac{54}{67}$.

b. the probability of rolling a number less than or equal to 10

Probability of rolling a number ≤ 10 = $\frac{63}{67}$.

(5) 5. Circle the correct response.

PROBABILITY LINE

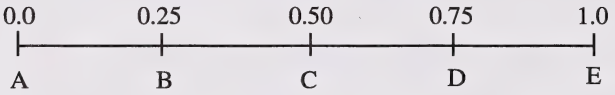


If an event is **certain** to occur, which probability represents this situation?

a. A
b. B
c. C
d. D
e. E

(5) 6. Circle the correct response.

PROBABILITY LINE



If the probability that an event will occur is equal to the probability that it will not occur, which position on the probability line represents this situation?

a. A
b. B
c. C
d. D
e. E

(5) 7. Suppose you are tossing a coin into the air and recording whether it lands heads (H) or tails (T). If you toss the coin thrice, what is the probability of recording HTT? Express your answer as a ratio in reduced fractional form.

There is only 1 way to get HTT.
There are 8 different outcomes.
(The order of tosses are important.)

$P(\text{HTT}) = \underline{\quad \mathbf{\frac{1}{8}} \quad}$

(5) 8. Two dice are tossed. What is the probability of tossing a number less than or equal to 10? Express your answer as a ratio in reduced fractional form.

There are 33 ways to get 10 or less.
There are 36 possible results on the dice.

$P(\text{number} \leq 10) = \frac{33}{36}$

$P(\text{number} \leq 10) = \underline{\quad \mathbf{\frac{11}{12}} \quad}$

(5) 9. A basket at the fruit stand contains 21 apples, 19 oranges, and 8 plums. If you were to close your eyes and reach into the basket to select one piece of fruit, what would be the probability of not purchasing a plum? Express your answers as a ratio reduced to lowest terms.

There are $21+19$ or 40 items that are not plums.
There are $21+19+8$ or 48 items in the basket.

$P(\text{not plum}) = \frac{40}{48}$

$P(\text{not plum}) = \frac{5}{6}$

There is a probability of $\frac{5}{6}$ of not selecting a plum.

(5) 10. Demographic Report for Huntsville

Age Class (a)	Number of People
0 - 9	156
10 - 19	194
20 - 29	152
30 - 39	155
40 - 49	124
50 - 59	107
60 - 69	74
70 +	38

While on a tour of Huntsville, suppose you said good-bye to the last person you saw. Expressing your answer in decimal form, determine the probability that the person is in one of the following categories.

a. older than 39 years of age

$$P(>39) = \frac{124+107+74+38}{1000}$$

$$= \frac{343}{1000} \text{ or } 0.343$$

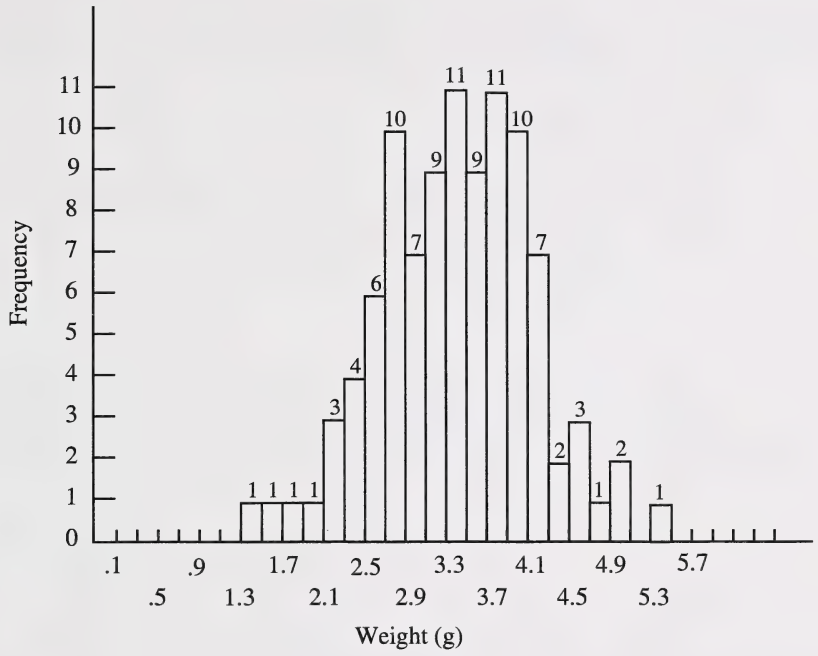
b. less than 50 years of age.

$$P(<50) = \frac{124+155+152+194+156}{1000}$$

$$= \frac{781}{1000} \text{ or } 0.781$$

6

11.



The horizontal axis of the above histogram represents the weights, in grams, of an assortment of chocolates for sale at The Olde Sweets Shoppe. Upon entering the store, you are unsure of which chocolate to purchase. You decided to blindly point in the direction of the display case to make your selection. Expressing your answers as reduced fractions or as decimals, determine the probability of choosing a chocolate weighing between the following weights.

a. 1.7 g and 2.5 g

$$P(1.7 < m < 2.5) = \frac{1+1+3+4}{100}$$

$$= \frac{9}{100} \text{ or } 0.09$$

b. 2.9 g and 3.7 g

$$P(2.9 < m < 3.7) = \frac{7+9+11+9}{100}$$

$$= \frac{9}{25} \text{ or } 0.36$$

c. 4.1 g and 5.7 g

$$P(4.1 < m < 5.7) = \frac{7+2+3+1+2+1}{100}$$

$$= \frac{4}{25} \text{ or } 0.16$$

(6) 12.

7	8	10	9
11	8	6	12
8	7	12	8
6	5	10	9
7	8	8	7
10	11	8	9
9	8	7	9
10	7	9	8

The above chart represents the shoe sizes of the students in Ms. Hunt's dance class. One student is selected at random to perform a tap dance. Expressing your answers as reduced fractions, determine the probability that the student's shoe size will be in one of the following categories.

a. size 9

$$P(9) = \frac{6}{32}$$
$$= \frac{3}{16}$$

b. either size 8 or size 9

$$P(8 \text{ or } 9) = \frac{15}{32}$$

c. not less than size 8

$$P(\geq 8) = \frac{23}{32}$$

(6) 13. As part of a compound event, three coins are tossed in the air and a record is made of how they land. Expressing your answers as reduced fractions, determine the probability of getting one of the following:

a. 2 tails

Desirable outcomes: HTT, THT, TTH

$P(2T) = \frac{3}{8}$ (There are 8 possible outcomes.)

b. 0 tails

Desirable outcomes: HHH

$P(0T) = \frac{1}{8}$

c. 3 heads

Desirable outcomes: HHH

$P(3H) = \frac{1}{8}$

(6) 14. As part of a compound event, three cards are drawn from a deck of 52 playing cards. Expressing your answers as reduced fractions, determine the probability of drawing one of the following combinations.

a. the ace ♣, king ♣, and queen ♣

$$P(A\clubsuit,\ K\clubsuit,\ Q\clubsuit) = \frac{3}{52} \times \frac{2}{51} \times \frac{1}{50}$$
$$= \frac{6}{132\,600}$$
$$= \frac{1}{22\,100}$$

b. the 7♠, 5♣, and 6♠

$$P(7\spadesuit,\ 5\clubsuit,\ 6\spadesuit) = \frac{3}{52} \times \frac{2}{51} \times \frac{1}{50}$$
$$= \frac{6}{132\,600}$$
$$= \frac{1}{22\,100}$$

(5) 15. Two items are selected at random, first from Table 1 and then from Table 2. Determine the following probabilities of the compound event. Report your answers as reduced fractions.

Table 1

4	2	4	1
1	3	1	2
3	2	1	2
2	3	4	1
1	2	4	4
1	3	4	1
1	2	4	2

Table 2

★	○	∞	Δ
∞	○	∞	★
★	Δ	★	Δ
★	○	★	○
★	∞	∞	★
∞	○	Δ	Δ
Δ	○	∞	∞

a. Probability (2 and ○)

$$P(2 \text{ and } ○) = \frac{8}{28} \times \frac{6}{28}$$
$$= \frac{48}{784}$$
$$= \frac{3}{49}$$

b. Probability (3 and Δ)

$$P(3 \text{ and } Δ) = \frac{4}{28} \times \frac{6}{28}$$
$$= \frac{24}{784}$$
$$= \frac{3}{98}$$

(6) 16. Loretta decides that she will simply guess at the answers to a 10-question true/false test. Before she writes the exam she simulates the situation using a coin and by letting **heads** represent a correct response. The following table is a summary of the outcomes of her simulation for the number of heads occurring per trial of 10 coin tosses.

Number of Heads	Number of Trials	Estimated Probability
0	2	$\frac{1}{72}$
1	3	$\frac{1}{48}$
2	9	$\frac{1}{16}$
3	17	$\frac{17}{144}$
4	28	$\frac{7}{36}$
5	34	$\frac{17}{72}$
6	26	$\frac{13}{72}$
7	13	$\frac{13}{144}$
8	7	$\frac{7}{144}$
9	5	$\frac{5}{144}$
10	0	0

Complete the above table and determine the probability that she will obtain the following scores. Express your answers as reduced fractions.

a. not less than 60%

$$P(\not< 60\%) = \frac{13}{72} + \frac{13}{144} + \frac{7}{144} + \frac{5}{144} + 0$$
$$= \frac{26+13+7+5+0}{144}$$
$$= \frac{17}{48}$$

b. between 30% and 70% (inclusive).

$$P(30\% \le s \le 70\%) = \frac{17}{144} + \frac{7}{36} + \frac{17}{72} + \frac{13}{72} + \frac{13}{144}$$
$$= \frac{17+28+34+26+13}{144}$$
$$= \frac{59}{72}$$

(5) 17. Circle the correct response.
Determine the most appropriate simulation for the following situation.
Mary has two pairs of shoes to choose from to wear to school. She randomly selects a pair of shoes each morning. What is the probability that she wears the same pair of shoes for a week?

A. Use a random number table considering odd and even numbers only.

B. Draw 2 numbers from a hat.

C. Toss a coin.

(D.) All of the above.

(5) 18. Circle the correct response.
Determine the most appropriate simulation for the following situation.
Mary has five pairs of shoes to choose from to wear to school. She randomly selects a pair of shoes each morning. What is the probability that she wears the same pair of shoes for six consecutive days?

(A.) Use a random number table of 6-digit numbers.

B. Roll a die.

C. Toss a coin 3 times.

D. All of the above.

(5) 19. In many countries, it is the son who takes care of the parents when they have reached their old age. Because of this, couples are inclined to continue to have children until they have a son.

a. Using this situation as the rule, do a simulation to find the following information.

i. the average number of children per family

Solutions will vary. This is one possible simulation.

B	B	GGGB	GB	B
GGGGB	GGB	GGB	B	GGB
B	GGGB	B	B	B
B	B	GGB	B	B
B	B	GB	GGGB	GB
GGB	GB	GGB	GB	GB
B	B	GGB	B	GB
GGGGGGGB	GGB	GGGB	GB	B
B	GB	B	GB	GGGB
GGB	B	B	GGGGB	GB

$\frac{109}{50}$ or about 2

There will be an average of 2 children per family.

ii. the average number of girls per family

Solutions will vary.

$\frac{59}{50}$ or about 1

There will be an average of 1 girl per family.

b. Would you expect more girls or more boys to be born? Explain your answer.

The answers will vary. Check to see how they support their answer.

Topic 1

_____ marks

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